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NASA TT F-672

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NASA TT F-672

PHYSICAL EXPERIMENTS WITH TEST BODIES

by V. B. Braginskiy

"Nauka" Press, Moscow, 1970

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1972

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Translation of "Fizicheskiye Eksperimenty s Probnymi Telami."
"Nauka" Press, Moscow, 1970

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the National Technical Information Service, Springfield, Virginia 22151
$3.00

FOREWORD

The subject of this book is physical experiments in which the detection of an effect can be reduced to the registry of a force or moment of force. Despite the fact that classical physics began precisely with such experiments, they are still being made. The reasons for formulating experiments are of an extremely basic nature (for example, the problem of detecting gravitational radiation, the search for quarks, etc.).

In the experiments carried out today or made during recent years, the attained sensitivity is extremely high and may astonish the experimental physicist who is not working in this field. On the other hand, the sensitivity in these experiments is increasing with each passing year. About 50 years ago Millikan discovered a single "excess" electron (or its absence) in a droplet in which the excess electron was accompanied by 10^{13} nucleons; now the same thing can be done "against a background" of 10^{18} nucleons. P. N. Lebedev, at approximately the same time, measured the pressure of a light flux with an intensity of about 1 W; now it is possible to measure the light pressure from fluxes of tens of microwatts. During 1959-1963 Dicke, in repeating the experiments of Eotvos in checking the equivalence principle, succeeded in increasing the sensitivity by three orders of magnitude. Evidently, in the future we can expect a further increase in sensitivity. The author has endeavored to describe the conditions (methodological and theoretical) necessary for increasing sensitivity, and also indicating the limits of resolution which are theoretically attainable. It has been found that these limits are to a certain extent similar to the recently discovered macroscopic quantum effects.

The book also includes descriptions of some recently performed interesting experiments involving the solution of fundamental physical problems and estimates of the limiting resolution in individual experiments discussed in the literature. The selection of material in the second part of the book was governed only by the importance of the physical problems which can be experimentally solved. Naturally, this selection was determined by the author's subjective point of view. Accordingly, the examples and illustrations in the second half of the book do not exhaust all the possible experiments in which it is important to detect small mechanical forces or moments of force.

The author takes this opportunity to express his appreciation for valuable comments made by Yu. L. Klimontovich, V. N. Rudenko and R. V. Khokhlov during the reading of the manuscript.

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PHYSICAL EXPERIMENTS WITH TEST BODIES

B. V. Braginskiy

ABSTRACT. This monograph is an examination of physical experiments in which the detection of an effect essentially involves the detection of a small force or moment of force acting on a macroscopic body (experiments with test bodies); the author analyzes the threshold response of a mechanical oscillator to exposure to a regular external force. The effects of light friction and radiometric oscillatory instability are examined. The optimum strategy for measuring a small regular force acting on a mechanical oscillator is described. Estimates of the minimum detectable magnetic field strengths, electric charge, acceleration, etc., are given. Experiments for checking the equivalence principle, for searching for elementary particles with a fractional electric charge, and for detecting quantum macroscopic effects are described. The prospects for detecting gravitational radiation and the possibilities of carrying out relativistic gravitational experiments in the nonwave zone are evaluated. The limiting attainable responses in experiments with test bodies in the search for relict quarks with a whole electric charge and electric dipole moments of elementary particles are examined. Methods are described for measuring small mechanical displacements and mechanical fluctuations in a space laboratory.

"Until we know why an elementary electric charge is identical in all processes, interest in fundamental problems in physics will continue unabated." -- W. Weisskopf. /9*

"Indeed, if something is unknown it is almost as if it does not exist." -- Apuleius, Metamorphoses, Book 10.

INTRODUCTION

The great number of experiments in which discovery of a physical effect has been reduced to registering a small force or moment of force acting on a

*Numbers in the margin indicate pagination in the foreign text.

macroscopic body have yielded fundamental physical information. For example, such experiments include those performed by Einstein and de Haas, Millikan, Eotvos, Dicke (checking the equivalence principle), Shubnikov and Lazarev (paramagnetism of nuclei), as well as a whole series of experiments proposed and in part carried out at the present time; mechanical experiments for the detection of parity nonconservation, search for rare particles with a fractional electric charge, for detecting gravitational radiation, etc. Relatively recently the effect of quantizing of a magnetic flux in superconducting cavities was discovered (quantum macroscopic effect). The discovery of this effect essentially involved the detection of a small mechanical moment of force.

With repetition of such experiments or carrying out of new experiments (it is convenient to refer to them as experiments with test bodies), the greatest interest is in the limiting attainable resolution. The development of modern experimental instrumentation has now made possible a substantial (by several orders of magnitude in comparison with already executed experiments) decrease in the friction connecting a test body to the laboratory, and /10
accordingly, decrease the fluxuation forces acting on a test body. Evidently, a major qualitative advance in reducing this fluctuation effect must be expected in formulating experiments with test bodies in space (in the presence of weightlessness, an intense vacuum, and in the absence of seismic interference). A considerable part of this monograph is devoted to an examination of the limiting resolution in experiments with test bodies. Emphasis is devoted to the fluctuation effects which theoretically cannot be eliminated.

In certain classical studies the minimum force applied to a mechanical oscillator, detectable against the background of thermal fluctuations, was usually computed for the case when the period of the oscillations τ_0 was approximately equal to the relaxation time τ^* (critical damping). The well-known expressions derived under this condition were clearly checked in the classical experiments made by Ising, Cernik, and others. These expressions have been incorporated into textbooks as an illustrative example of the limiting response of galvanometers, electrometers, etc. It follows from the condition $\tau_0 \simeq \tau^*$ that in order to increase sensitivity (response) in experiments with test bodies, the only methods are an increase in τ_0 and τ^* and a repetition of the number of measurements. On the other hand, an increase in τ^* means a decrease in the friction coefficient H, being a source of the fluctuation force, whereas the period τ_0, in general, is unrelated to the sources of fluctuation forces.

Thus, it is clear from such simple qualitative considerations that increases in sensitivity can be achieved only by increasing τ^*. In this way the experimenter will be concerned with a quasiconservative system; if the test body is rigidly connected to the laboratory, for such an oscillator $\tau_0/\tau^* \ll 1$.

In all the experiments mentioned above, as well as in most of those discussed in the literature, it is usually assumed that the force $F(\tau)$, whose effect on a test body must be detected, can be coded. This means that in formulating experiments there is no need to limit oneself to quasistatic measurements, for which, as is well known, it is recommended that $\tau_0 \simeq \tau^*$ be /11
selected. Preliminary information on the $F(\tau)$ form requires solution of the problem of the optimum methods for detecting the response of a mechanical system to the $F(\tau)$ effect. Since a decrease in the thermal fluctuation effect requires an increase in the τ^* value, with a sufficiently high experimental skill, the time expended on measurements τ_{meas} and $\hat{\tau}$, the time of the $F(\tau)$ effect, should be substantially less than the relaxation time τ^*. In other words, the optimum detection of a response to $F(\tau)$ should in general be in a nonequilibrium system.

The vigorous development of the theory of discrimination of a signal from noise during the 1940's and 1950's was for the most part associated with an optimizing of operations performed in a receiver on the sum (signal plus noise) with a statistically effective use of preliminary information on the signal. This theory was developed due to the needs for developing distant communications and radar, and therefore the relaxation time in the receiver converting the sum (signal plus noise) was considered small, so that the processes occurring in it could be considered close to equilibrium. Evidently, no detailed analysis was made for optimum detection for cases $\tau_{meas}/\tau^* \ll 1$ and $\hat{\tau}/\tau^* \ll 1$.

The literature contains a relatively large number of examinations of the possibility of observing physical effects involving detection of the force $F(\tau)$ (or the moments of force) acting upon a mechanical oscillator. In many cases, in such a study the threshold value of the detectable parameter $[F(\tau)]_{min}$ was determined from the equilibrium value of oscillator energy $[F(\tau)]_{min} \simeq \sqrt{\kappa T m \omega_0^2}$.

Obviously, such an estimate is inadmissible, since such an important parameter as the ratio between the times τ_{meas} (or $\hat{\tau}$) and the relaxation time τ^* was not included.

In summarizing the preliminary results of the qualitative considerations presented above, it can be concluded that an increase in the τ^* value, dependent essentially only on the "culture" of the experiment, should, in the case of thermal fluctuations, result in a decrease in the threshold of detectable $F(\tau)$ effects on a test body, assuming that these effects are limited in time. Unquestionably, a rigorous examination of the problem of detection of a weak effect on a mechanical oscillator or free body, in the /12 case of large τ^*, presents no difficulties, at least in the classical approximation. This problem can be solved, for example, using known expressions derived by Chandrasekhar for the Langevin equation [1]. However, in the studies by Chandrasekhar the case $\hat{\tau}/\tau^* \ll 1$ was not analyzed in detail.

A decrease in the $\hat{\tau}/\tau^*$ value should have the following result: the detectable quantities of energy ΔW imparted to a test body or removed from it by the force $F(\tau)$ should become substantially less than the equilibrium energy value κT (for the selected degree of freedom). This means that there is a limit of applicablity of the classical solution of this problem for some quite small $\hat{\tau}/\tau^*$. In addition, in the classical examination no allowance is usually made for the fluctuation effect of the device registering small displacements of a test body. In the case of a quite small $\hat{\tau}/\tau^*$ this influence is decisive. In terrestrial laboratories, torsion balances on thin quartz filaments make it possible to obtain $\tau^* \simeq 10^6$ sec. In centrifuges on magnetic suspensions with a servosystem, as well as in gyroscopes with electrostatic servo-suspensions with a vacuum of about 10^{-8} mm Hg the τ^* value attains about 10^9 sec (Beams, Nordoik). Thus, even under terrestrial conditions it is possible to obtain a quite small $\hat{\tau}/\tau^*$ ratio.

The first chapter is devoted to an examination of a number of problems involved in detecting a small force acting on a test body. This chapter includes the classical examination of detecting a small regular (limited in time and of a known form) force acting on a mechanical oscillator (§ 1). This section also discusses the methods for detecting the response to a force when $\hat{\tau}/\tau^* \ll 1$ against a background of fluctuations, and gives illustrative examples.

Analysis of the dynamic effect of radio engineering and optical indicators on a mechanical oscillator is presented in § 2. This section examines the effect exerted on the period τ_0 and the time τ^* by the device used in registering small displacements; it describes the appearance of oscillatory instability characteristic for a definite type of indicators, and gives an analysis of the dissipative influence of optical radiation on mechanical motion, which is comparable to the friction introduced by a rarified gas in a deep vacuum. /13

Section § 3 discusses the situation, most important with respect to limiting response, when the τ^* value is so great that the fluctuation effect on the test body by the small displacement indicator determines the limiting attainable response. Analysis of this case shows that there is an optimum method for selecting the indicator parameters so that the minimum detectable force (or moment of force) is not dependent on the indicator properties. The analytical expressions derived here make it possible to compare the significance for limiting sensitivity of classical thermal fluctuations (with the smallness of $\hat{\tau}/\tau^*$ taken into account) and quantum fluctuations in the indicator.

Section 4 is devoted to a discussion of the optimum strategy in experiments with test bodies, comparison of the well-known classical expressions and the derived analytical expressions for optimum indicators. This section also gives numerical estimates for the attainable response (electrometers, accelerometers, magnetometers, etc.).

The second chapter describes the methods employed in experiments performed during recent years and having fundamental importance for basic physical concepts.

Section 5 describes an experiment for checking the equivalence principle (Dicke); section 6 describes an experiment in which quantizing of a magnetic flux in superconductors was discovered (Fairbank, Doll, Näbauer and Deaver); section 7 describes experiments for detecting rare particles with a fractional electric charge (quarks). In each of the sections there is a comparison of the resulting resolution with the theoretically attainable response on the basis of the "optimum strategy" discussed in the first chapter.

The third chapter is devoted to the possibilities of detecting gravitational radiation (§ 8) and an analysis of the possibilities of carrying out /14 some relativistic gravitational experiments in the nonwave zone (§ 9). In section 8 there is also a short review of present-day concepts concerning sources of gravitational radiation of extraterrestrial origin. Section 10 discusses experiments in which it is possible to detect new properties of elementary particles.

The Appendix contains a brief review of methods for detecting small mechanical displacements (§ 11), as well as some information based on measurements carried out recently for determining the level of quasiseismic fluctuations which will be observed for orbital space laboratories (§ 12).

CHAPTER I /15

DETECTION OF SMALL FORCES ACTING ON A MECHANICAL OSCILLATOR

§ 1. Oscillatory System With a Large Time Constant Experiencing the Effect of a Fluctuating Force. Methods for Detecting a Small Regular Force.

We will examine a very simple model of an experiment with a test body. Visualize that it is necessary to detect the effect of a force $F\ (\tau)$ on a mass m which is connected to a laboratory by the rigidity K, having a dissipation corresponding to the friction coefficient H. We will assume that preliminary information is available concerning the form of the $F\ (\tau)$ force (regular effect). The following cases are most frequently encountered: $F\ (\tau)$ has the form of a sinusoidal train or $F\ (\tau)$ is a single impulse. Henceforth we will be concerned only with the level of attainable sensitivity (response) in detecting regular $F\ (\tau)$ effects on such an oscillator, and we will not be concerned with quasistatic measurements in which the response level is determined by drift characteristics (temperature stability, stability of the rigidity element, etc.).

We will assume that in addition to $F\ (\tau)$, the oscillator mass is acted upon by a stationary fluctuation force F_{fl}. In the special case of thermal fluctuations the spectral density F_{fl} is equal to $(F_{fl})^2_\omega = 4\kappa TH$, where κ is the Boltzmann constant and T is temperature. Noise of nonthermal origin can /16 be avoided by using a number of technical methods (antiseismic platforms, acoustic shielding, etc.). However, in theory the thermal fluctuations of a mechanical oscillator cannot be eliminated. This circumstance is usually emphasized in determining the limiting response of galvanometers, electroscopes, etc. However, in this case the level of attainable response is related to the rather special case of measurements when the time expended on measurements is about equal to the time of oscillator damping (for example, see [2, 3]). If one does not limit himself to this case, as will be seen from the text which follows, the level of attainable response is substantially higher and the analytical expressions determining the minimum detectable $F\ (\tau)$ value will be different.

If it were possible to eliminate nonthermal fluctuation effects on the mass m, it would be possible to assert immediately that the approximate condition for detecting $F(\tau)$ has the form

$$F(\tau) \gtrsim \sqrt{4\varkappa T H \Delta f}, \tag{1.1}$$

where Δf is the frequency band within which the greater part of the $F(\tau)$ spectrum falls. It can be seen from this condition that in order to increase the threshold response it is necessary to decrease T and H. The Δf value is determined by the $F(\tau)$ form. Condition (1.1) is approximate since the method for measuring the oscillator response to $F(\tau)$ was not indicated.

We will assume that we have an ideal instrument registering mechanical movements (as small as desired) under the influence of $F(\tau) + F_{fl}$ and not making any contribution to F_{fl}. Such an assumption is possible within the framework of the classical problem. Somewhat later (§ 3) we will examine the fluctuation (classical and quantum) effect of such an indicator on an oscillator and will examine the optimum measurement strategy.

The simplest method for detecting response to the $F(\tau)$ effect is to register the change in amplitude of oscillator oscillations. As already mentioned in the Introduction, a decrease in H, necessary for increasing response, has the following effect: the relaxation time $\tau^* = 2m/H$ is substantially greater than τ_{meas}, the reasonable time which can be expended on /17 measurement, and than $\hat{\tau}$, the time of the $F(\tau)$ effect. Thus, when $\hat{\tau}/\tau^* \ll 1$ and $\tau_{meas}/\tau^* \ll 1$ the oscillator will behave as a system close to conservative, and the amplitude of its oscillations will be a slowly varying function (for example, see [4, 5]):

$$x(\tau) = A(\tau)\sin[\omega_0\tau + \varphi(\tau)], \qquad \overline{A(\tau)} \gg \frac{1}{\omega_0}\frac{\partial A(\tau)}{\partial\tau}. \tag{1.2}$$

In order to assert with some predetermined reliability that in addition to the stationary fluctuation force F_{fl}, the mass m will be acted upon by the force $F(\tau)$ during some interval $\hat{\tau}$, it is necessary to determine the limits within which it is possible to change the amplitude of oscillator oscillations under the influence of F_{fl}: $[A(\tau) - A(0)]_{1-\alpha}$. These limits are determined only with the stipulated degree of probability $(1 - \alpha)$; the α value is usually called the statistical error of the first kind. Thus, by finding these limits (in mathematical statistics they are sometimes called quantiles [6]), it will be possible to determine the threshold response for $F(\tau)$.

The expression for the probability density of an arbitrary distribution of amplitude of oscillations $A(\tau)$ at the time $\hat{\tau}$ has the form [5]

$$p[A(\hat{\tau}) | A(0)] = \frac{A(\hat{\tau})}{\sigma^2 (1 - G^2)} I_0 \left(\frac{G A(0) A(\hat{\tau})}{\sigma^2 (1 - G^2)} \right) \exp \left[- \frac{A^2(\hat{\tau}) + G A^2(0)}{2\sigma^2 (1 - G^2)} \right], \tag{1.3}$$

where for our case

$$G \simeq e^{-|\hat{\tau}|/\tau^*}, \qquad \sigma^2 = \overline{[A(\tau)]^2}.$$

The $[A(\hat{\tau}) - A(0)]_{1-\alpha}$ value, in accordance with the definition of $p[A(\tau)| A(0)]$ can be found by solving the equation

$$1 - \alpha = \int\limits_{A(0)}^{[A(\hat{\tau})]} p[A(\tau) | A(0)] \, dA(\hat{\tau}). \tag{1.4}$$

We will examine two cases: $A(0) = 0$ and $A(0) \simeq \sigma$. In the first case /18
equation (1.4) has the simple form:

$$1 - \alpha = 1 - \exp \left[- \frac{[A(\hat{\tau})]^2_{1-\alpha}}{2c\sigma^2} \right], \qquad c = \frac{2\hat{\tau}}{\tau^*} \ll 1, \tag{1.5}$$

hence

$$[A(\hat{\tau})]_{1-\alpha} = \sigma \sqrt{\frac{2\hat{\tau}}{\tau^*}} \cdot \sqrt{2 \ln (1/\alpha)}. \tag{1.6}$$

The $[A\ (\hat{\tau})]_{1-\alpha}$ value does not vary greatly for initial values $A\ (0) \lesssim \sigma\sqrt{c}$, and by using expression (1.6) it is possible, by stipulating α, to estimate with the probability $(1 - \alpha)$ the limit of possible change in the amplitude of oscillations $[A\ (\hat{\tau}) - A\ (0)]_{1-\alpha}$ with time $\hat{\tau}$. It can be seen from (1.6) that this requires a knowledge of $\hat{\tau}$, τ^* and σ. In the case of thermal fluctuations

$$K\sigma^2 = \varkappa T.$$

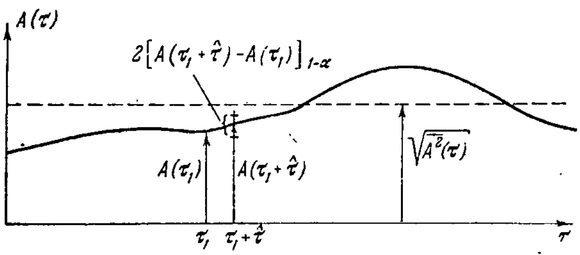

Figure 1

Thus, if the oscillator has the small initial amplitude $A\ (0) = 0$ or $A\ (0) \lesssim \sigma\sqrt{c}$) and with the course of time $\hat{\tau}$ the amplitude of its oscillations exceeds the value (1.6), with the reliability $(1 - \alpha)$ it will be possible to assert that in addition to F_{fl} the oscillator will be acted upon by some additional force $F(\tau)$ (Figure 1).

As can be seen from the above, we obtained only a threshold expression, the ordinary expression for detection theory. In other words, except for "yes" or "no" nothing can be said concerning the additional effect $F\ (\tau)$ if $A\ (\hat{\tau})$ insignificantly exceeds or does not exceed the limit $[A\ (\hat{\tau}) - A\ (0)]_{1-\alpha}$. /19

Thus, the $[A\ (\hat{\tau}) - A(0)]_{1-\alpha}$ value corresponds to the threshold response when detecting the small parameter $F\ (\tau)$. It is clear that in order to obtain a relative accuracy of about 10%, for example, it is necessary that during the time $\hat{\tau}$ the $F\ (\tau)$ effect imparts to the oscillator an amplitude of oscillations $A\ (\hat{\tau})$ approximately 10 times greater than $[A\ (\hat{\tau}) - A\ (0)]_{1-\alpha}$.

We will now return to the situation when $A\ (\hat{\tau})$ is close to the limiting value. The interference of oscillations caused by $F\ (\tau)$ and F_{fl} can have the following effect: $A\ (\hat{\tau})$ does not exceed $[A\ (\hat{\tau}) - A\ (0)]_{1-\alpha}$, and using the criterion described above, it will be necessary to draw the incorrect conclusion that $F(\tau) = 0$. In order to estimate the probability of such an outcome it is necessary to introduce the statistical error of the second kind β, the probability that on the assumption $F(\tau) \neq 0$ one will obtain the result $A(\tau) < [A(\hat{\tau}) - A(0)]_{1-\alpha}$.

We will assume that the $F(\tau)$ force, in the absence of F_{fl}, can sway the oscillator during the time $\hat{\tau}$ to the amplitude B. We will stipulate some value $\zeta = B/[A(\hat{\tau}) - A(0)]_{1-\alpha}$ and compute the β value. We will limit ourselves here to the case $\zeta \geqslant 1$ and we will assume that the phase shift between the oscillations caused by $F(\tau)$ and F_{fl} is random. In addition, we will assume that $A(0) = 0$. It is easy to demonstrate, taking into account the interference of oscillations caused by $F(\tau)$ and F_{fl}, that

$$\beta = \frac{1}{\pi}\int_{\varphi_1}^{\pi} d\varphi \int_{-B\cos\varphi - \sqrt{[A(\tau)]^2_{1-\alpha} - B^2\sin^2\varphi}}^{-B\cos\varphi + \sqrt{[A(\tau)]^2_{1-\alpha} - B^2\sin^2\varphi}} p[A(\tau)|0]\, dA(\tau), \tag{1.7}$$

where $\sin\phi_1 = 1/\zeta$.

Replacing the notation

$$[a(\hat{\tau})]_{1-\alpha} = \frac{[A(\hat{\tau})]_{1-\alpha}}{\sigma}$$

and taking into account that $\zeta = B/[A(\hat{\tau})]_{1-\alpha}$, we obtain /20

$$\beta = \frac{1}{\pi}\int_{\varphi_1}^{\pi} \left\{\exp\left[-\frac{[a(\hat{\tau})]^2_{1-\alpha}(-\zeta\cos\varphi - \sqrt{1-\zeta^2\sin^2\varphi})}{2c}\right] - \right.$$
$$\left. - \exp\left[-\frac{[a(\hat{\tau})]^2_{1-\alpha}(-\zeta\cos\varphi + \sqrt{1-\zeta^2\sin^2\varphi})}{2c}\right]\right\} d\varphi, \tag{1.8}$$

where, like above, $c = 2\hat{\tau}/\tau^*$.

The $[a(\hat{\tau})]_{1-\alpha}$ value is related to the magnitude of the error of the first kind α by expression (1.6). Using it, it is possible to express β as a function of α and ζ:

$$\beta = \frac{1}{\pi}\int\limits_{\varphi_1}^{\pi}\{\alpha^{[-\zeta\cos\varphi - \sqrt{1-\zeta^2\sin^2\varphi}]} - \alpha^{[-\zeta\cos\varphi + \sqrt{1-\zeta^2\sin^2\varphi}]}\}\,d\varphi. \qquad (1.9)$$

When $\alpha = 0.05$ and $\zeta = 1.4$ we have $\beta = 0.117$; when $\alpha = 0.05$ and $\zeta = 2$, the β value is 0.005.

Thus, using the boundary value (1.6), the probability of not detecting the effect of the $F(\tau)$ force, which in the absence of F_{fl} causes the amplitude $B = 2[A(\hat{\tau})]_{1-\alpha}$, does not exceed 0.5%. The β parameter does not substantially change if $A(0) \neq 0$, but does not exceed $\sigma\sqrt{c}$.

Now we will return to the case when $A(0)$ and $A(\tau)$ are not small in comparison with the σ value. We will determine the limiting values $[A(\hat{\tau}) - A(0)]_{1-\alpha}$, when $A(0) \simeq \sigma$. Taking into account that $c = 2\hat{\tau}/\tau^* \ll 1$, the function $I_0(x)$ in (1.3) can be replaced by its asymptotic expression for large values of the argument

$$I_0(x) \simeq (2\pi x)^{-1/2}e^x.$$

In this case the expression for the probability density of an arbitrary distribution of amplitudes (1.3) assumes the simple form:

$$p[a(\hat{\tau})\,|\,a(0)] \cong \sqrt{\frac{a(\hat{\tau})}{a(0)}}\frac{1}{\sqrt{2\pi c}}\exp\left[-\frac{[a(\hat{\tau}) - a(0)]^2}{2c}\right], \qquad (1.10)$$

where $a(\hat{\tau}) = A(\hat{\tau})/\sigma$, $a(0) = A(0)/\sigma$.

Thus, when $A(0) \simeq \sigma$ and with small $\hat{\tau}$ the normalized change in amplitude $[a(\tau) - a(0)]$ asymptotically conforms to a normal law with a dispersion $c = 2\hat{\tau}/\tau^*$. /21

Accordingly, in order to find the limiting value $[a(\hat{\tau}) - a(0)]_{1-\alpha}$, which limits the possible changes in a with time $\hat{\tau}$ with some probability of error of the first kind α, it is possible to use the ordinary procedure for a normal distribution.

$$\alpha = [1 - \Phi(u_{1-\alpha})], \qquad (1.11)$$

where

$$\Phi(u_{1-\alpha}) = \frac{2}{\sqrt{2\pi}} \int_0^{u_{1-\alpha}} \exp\left(-\frac{\xi^2}{2}\right) d\xi;$$

$$u_{1-\alpha} = \frac{[a(\hat{\tau}) - a(0)]_{1-\alpha}}{\sqrt{c}}.$$

Stipulating α and using well-known tables for Φ $(u_{1-\alpha})$, it is possible to obtain limiting values $[A(\tau) - A(0)]_{1-\alpha}$. For example, when $\alpha = 0.05$, $u_{0.95} = 1.96$.

$$[A(\hat{\tau}) - A(0)]_{0.95} = \sigma\sqrt{c}\cdot 1.96;$$

in the general case

$$[A(\hat{\tau}) - A(0)]_{1-\alpha} = \sigma\sqrt{c}\, u_{1-\alpha}. \qquad (1.12)$$

It is interesting to compare the close functions $[2 \ln (1/\alpha)]^{1/2}$ and $u_{1-\alpha}$. Table 1 gives the numerical values of these functions for several α values. The table shows that the numerical values of these two functions differ insignificantly when $0.05 < \alpha < 0.0001$. Thus, the limiting value for the change in amplitude is not critical for the initial value of the amplitude of oscillations. /22

TABLE 1

α	$[2 \ln (1/\alpha)]^{1/2}$	$u_{1-\alpha}$
0.05	2.45	1.96
0.01	3.04	2.58
0.005	3.25	2.81
0.001	3.72	3.29
0.0005	3.90	3.48
0.0001	4.29	3.88

The magnitude of the error of the second kind β for the case $A \simeq \sigma$ can be computed as is usually done in the case of a normal distribution (for example, see [6]).

As can be seen from expressions (1.6) and (1.12), the limiting value $[A(\hat{\tau}) - A(0)]_{1-\alpha}$, determining the minimum detectable change in amplitude caused by the external effect $F(\tau)$ during the time $\hat{\tau}$, is equal to the product $\sigma\sqrt{2\tau/\tau^*}$, multiplied by a numerical factor of about 2-4, dependent on the selected values of the statistical errors of the first and second kinds α and β. In other words, for determining $[A(\hat{\tau}) - A(0)]_{1-\alpha}$, and accordingly, for determining $[F(\tau)]_{min}$ it is necessary to know the σ, τ and τ^* values. As pointed out in the Introduction, the τ^* value, even for ordinary torsional pendulums, is ten days ($\tau^* \sim 10^6$ sec), whereas for determining the σ value a time of at least $3\tau^*$ is required. However, if it is possible to make some measurements of identical $F(\tau)$ effects with an identical effect time of τ (repetition of measurements) or if when $\tau_{meas} > \hat{\tau}$ it is possible to determine the change in amplitude of oscillations several times during time intervals of $\hat{\tau}$, the requirement for preliminary information on the σ and τ^* values disappears. In this case in place of $\sigma\sqrt{c}\, u_{1-\alpha}$ it is necessary to take $s(n-1)^{-1/2}t_{1-\alpha}$, where s^2 is the estimate of the dispersion $\sigma^2 c$ in change of amplitude during the time $\hat{\tau}$, determined experimentally, n is the number of repetitions, and $t_{1-\alpha}$ is the quantile of Student's t [6]. Already when $n = 10$ the quantile $t_{1-\alpha}(n) \simeq u_{1-\alpha}$, and s^2 can deviate from $\sigma^2 c$ by not more than 30-40%. This procedure, as follows from the above, is correct only for the case $A(0) \simeq \sigma$, i.e., when the deviation in amplitude of the oscillator is close to the normal law, and is not suitable for $A(0) \lesssim \sigma\sqrt{c}$. However, it is easy to show that if $A(0) > \sigma$ (for example, $A(0) \sim 3\sigma - 5\sigma$) and the measurement of deviations $A(\tau) - A(0)$ is performed with a correction for the monotonic decrease (reading from the regression line), the asymptotic normality /23
of the random deviations is retained.

Thus, in theoretical predictions in experiments it is possible to use expressions (1.6) and (1.12), but the described procedure can be used in the case of direct measurements. At the end of this section we will give examples illustrating the application of these methods in detecting small effects on mechanical oscillators.

Now we will note an important condition necessary for satisfying the mentioned methods for discriminating a signal from noise. If σ and τ^* are unknown, the experimenter must be able to repeat the measurement several times[1], or be able, without the F (τ) effect, to determine s^2, the evaluation $\sigma^2 c$, i.e., it is necessary that the time τ_{meas}, expended on the measurement, exceed $\hat{\tau}$.

Now we will discuss the physical corollaries from the expressions (1.6) and (1.12) derived above. As can be seen from these expressions, the minimum change in the amplitude of oscillator oscillations which can still be detected is dependent on the $\hat{\tau}$ and τ^* parameters, but in the case of thermal fluctuations, on friction in the oscillator. The quantity of energy which is imparted to or drawn from the oscillator by a regular external effect which can be distinguished in accordance with (1.6) and (1.22), is:

when A $(0) = 0$

$$\Delta W = K\{\zeta(\alpha,\ \beta)[A(\hat{\tau})]_{1-\alpha}\}^2 = 4\ln(1/\alpha)\{\zeta(\alpha,\ \beta)\}^2 \varkappa T \frac{\hat{\tau}}{\tau^*}; \qquad (1.13)$$

when A $(0) \simeq \sigma$

$$\Delta W = 2K\sigma\zeta'(\alpha,\ \beta)[A(\hat{\tau}) - A(0)]_{1-\alpha} = 2\sqrt{2}\zeta'(\alpha,\ \beta)u_{1-\alpha}\varkappa T\sqrt{\frac{\hat{\tau}}{\tau^*}}. \qquad (1.14)$$

In (1.13) and (1.14) $K = m\omega_0^2$ is oscillator rigidity.

---

[1] The number n of repetitions must satisfy the condition $n \geqslant 2$, since the $t_{1-\alpha}$ (n) value was determined with the number of statistical degrees of freedom $f \geqslant 1$; $f = 1$ corresponds to two independent measurements.

As can be seen from (1.13) and (1.14), the different quantities of energy constitute fractions of the equilibrium value κT. These fractions are the lesser the smaller the $\hat{\tau}/\tau^*$ ratio. This result is not surprising, if it is taken into account that when $\tau^* \gg \hat{\tau}$ we are dealing with a nonequilibrium process. The oscillator, in thermal equilibrium with the laboratory, has a mean energy κT. During the time τ^* the oscillator energy changes by a value of about κT, and during the time $\hat{\tau}$, by a value which is the smaller the lesser the $\hat{\tau}/\tau^*$ ratio. The factors on κT $(\hat{\tau}/\tau^*)$ and $\kappa T \sqrt{\hat{\tau}/\tau^*}$ in (1.13) and (1.14), as follows from what has been said above, are of the order of several units. If there is no need to take into account statistical errors of the second kind, only $4 \ln (1/\alpha)$ and $2\sqrt{2}u_{1-\alpha}$ respectively remain in the factors. /24

Now we will find the minimum value of the force effect $F(\tau)$ on a mechanical oscillator detectable using the procedure described above. We will examine the very simple case when $F(\tau) = F_0 \sin \omega\tau$ during the interval $0 \leqslant \tau \leqslant \hat{\tau}$ and $F(\tau) = 0$ outside this interval. We will also assume that $\omega = \omega_0$. If the initial $F(\tau)$ phase is selected in accordance with the instantaneous phase value of the oscillator oscillations, the change in the amplitude of the oscillations is $B \cong F_0\hat{\tau}(2m\omega_0)^{-1}$, since $\hat{\tau} \ll \tau^*$, and B can be computed as for the conservative system. Hence, requiring that B be not less than $[A(\hat{\tau})] - A(0)]_{1-\alpha}$, for the case of thermal fluctuations we obtain

$$[F_0]_{\min} = \theta\sqrt{\frac{2\kappa TH}{\hat{\tau}}} = \theta\sqrt{\frac{4\kappa Tm}{\hat{\tau}\tau^*}}, \tag{1.15}$$

where θ is equal to $\zeta(\alpha, \beta)\sqrt{2 \ln (1/\alpha)}$ when $A(0) \lesssim \sigma\sqrt{e}$, or $\zeta'(\alpha, \beta)\, u_{1-\alpha}$ when $A(0) \simeq \sigma$.

Expression (1.15) for a case when $F(\tau)$ has the form of a train of sinusoidal oscillations, gives the precise value for the threshold amplitude of the $[F_0]_{min}$ force in the case of known T, m, τ^*, $\hat{\tau}$, α and β and with the selected measurement method (measurement of change in the amplitude of oscillations). It can also serve as an evaluation in theoretical predictions of experimental results, since in measurements the $\sigma \sqrt{c} u_{1-\alpha}$ parameter is replaced by the close parameter $s\ (n\ -1)^{-1/2}\ t_{1-\alpha}\ (n)$ (see above).

As might be expected, the precise expression (1.15) differs only by a numerical factor from the approximate expression (1.1). It follows from (1.15) that in order to increase the threshold response it is necessary to increase both τ^* and $\hat{\tau}$. Since F_0 decreases as $(\tau)^{-1/2}$, an increase in the duration of the train and an increase in the number n of repetitions of the measurements identically decrease the $[F_0]_{min}$ value. /25

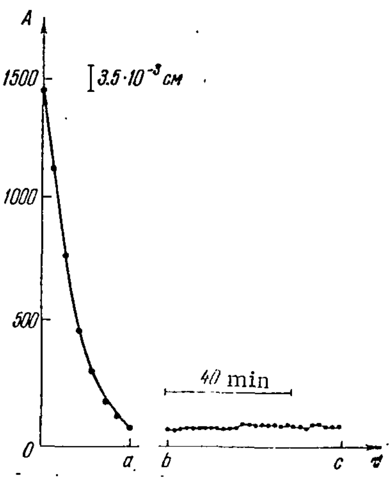

Figure 2

The correctness of expressions (1.13), (1.14), and (1.15) is limited: a) by the classical approach to the problem and b) the fact that no allowance was made for the fluctuation effect of the indicator of oscillator small oscillations. In the case of sufficiently large τ^* (sufficiently small H), this effect should be felt. The effects associated with this influence are examined in detail in sections 2 and 3.

In concluding this section we will discuss examples illustrating the examined procedure for detecting a weak effect on a mechanical oscillator with large τ^*.

Figure 2 shows a record of the amplitudes of oscillations of a horizontal torsional pendulum with the period τ_0 = 230 sec and a time constant $\tau^* \gtrsim 10^5$ sec. The pendulum is a dumbbell with the mass m = 25 g on the ends, suspended on a tungsten filament 100 μ in diameter. The entire pendulum was placed in a vacuum housing (for further details see [7]). In the time interval from 0 to a the pendulum is forcefully damped. This is easily done using several force impulses (for example, electrostatic or gravitational), applied in the necessary phase of oscillations to the dumbbell, since when $\hat{\tau} \ll \tau^*$ the phase of pendulum oscillations changes slowly with time. The time interval from a to b is 1.5 hours. A highly sensitive capacitive transducer was used as an indicator of small displacements. /26

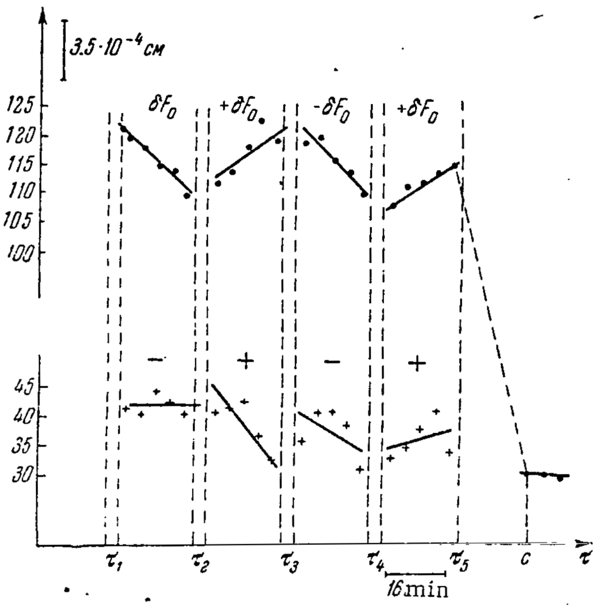

Figure 3

Figure 3 shows a record of oscillations of this same pendulum with a large magnification. The upper part of the figure shows a record of the change in amplitude of pendulum oscillations under the influence of fluctuations and sinusoidal trains with the amplitude $F_0 \simeq$ $\simeq 5 \cdot 10^{-7}$ dyne and a duration of 5 pendulum oscillation periods. The phase of the oscillations in these trains was selected in such a way that in the intervals τ_1-τ_2 and τ_3-τ_4 the force damped the pendulum, whereas in the intervals τ_2-τ_3 and τ_4-τ_5 it swayed the pendulum. It is easy to see the difference in the slopes of the regression lines drawn through the points corresponding to the maximum displacements of the dumbbell. The lower part of the figure shows a record of the amplitudes of oscillations of this pendulum in the absence of a regular external effect. It was more convenient to compare the difference in the slopes of the re- /27 gression lines, rather than the difference between the amplitudes at the beginning and at the end of the selected time intervals, since this made it possible to reduce the contribution of fluctuations created by the capacitive transducer. A statistical comparison of the two groups of regression line slopes (with the force swaying and damping the pendulum) can be made using Student's t, which corresponds to the recommendations presented above. When

$\hat{\tau}$ = 1,500 sec and τ_{meas} = 4 hours, it was possible to resolve the amplitude of the force $[F_0]_{0.95} = 1.2 \cdot 10^{-7}$ dyne with a reliability 0.95. We note that in these measurements no antiseismic shielding was used, and therefore the resolution level was determined by nonthermal noise. The method used in this experiment was described in greater detail in [7].

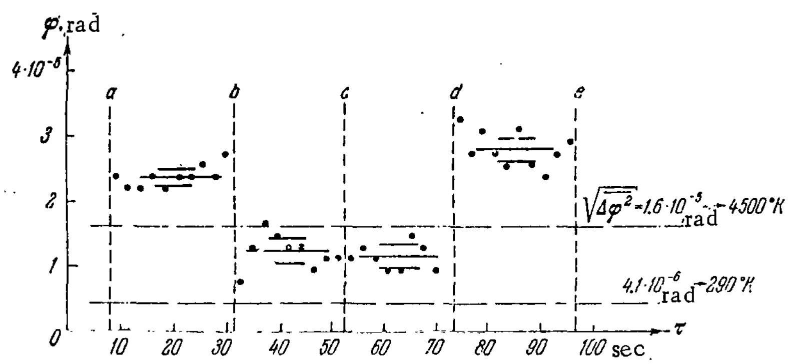

Figure 4

Figure 4 shows a record of the amplitudes of oscillation of a light torsional pendulum which was used as a ponderomotive indicator of the intensity of light radiation (for further details see [8], [9]). The pendulum was a glass plate measuring 1 × 0.3 × 0.01 cm, suspended on a tungsten filament 6 μ in diameter and 4 cm in length. The plate was coated with a layer of silver about 10 μ thick. The pendulum, mounted in a sealed flask, had a period of torsional oscillations τ_0 = 2.3 sec and a relaxation time τ^* = = 40 min. The torsional rigidity of the filament was $K_\phi = 2.4 \cdot 10^{-3}$ dyne·cm. /28
A photoelectric amplifier was used in registering small torsional oscillations and this made it possible to register small torsional oscillations with a response threshold of $8 \cdot 10^{-7}$ rad per oscillation. The pendulum was not placed on an antiseismic platform and therefore the mean square value of the deflection angle was large $\sqrt{\overline{\Delta\phi^2}} = 1.6 \cdot 10^{-5}$ rad. This value corresponds to

a relatively high equivalent noise temperature $T_{equiv} = K_\phi \overline{\Delta\phi^2}/\kappa = 4{,}500°K$. However, even with such a relatively high T_{equiv} value, during the time $\tau = 23$ sec (ten periods) it was possible to resolve small changes in pendulum energy. The minimum quantities of energy were about $\Delta W = \kappa 60°K = 7.8 \cdot 10^{-15}$ erg. Figure 4 shows 38 values of the angular amplitudes of pendulum oscillations, registered in a series. At the times *b* and *d* the pendulum was damped and swayed by pressure of a short light impulse of energy $0.9 \cdot 10^3$ erg (this corresponds to an impulse moment of $3 \cdot 10^{-8}$ dyne•cm•sec). In the figure the horizontal lines denote the mean angular amplitudes of oscillations and the confidence limits for them (with a probability level 0.95) in the time intervals *a-b, b-c, c-d,* and *d-e*. In the intervals *b-c* and *c-d* the mean amplitudes are statistically indistinguishable. The difference could be considered significant if the means differed by more than a half-width of the confidence interval $\Delta\phi_{con}$.

In our case $\Delta\phi_{con} \cong 1.8 \cdot 10^{-6}$ rad. This means that by swaying the pendulum during the time $\hat{\tau} = 23$ sec (or less) from small amplitudes to $\Delta\phi = 1.8 \cdot 10^{-6}$ rad it is possible to register the energy input $\Delta W = K_\phi (\Delta\phi_{con})^2 = 7.8 \cdot 10^{-15}$ erg $= \kappa 60$ °K. This value agrees satisfactorily with the estimate of the minimum detectable quantity of energy, assuming as a point of departure that $T_{equiv} = 4{,}500$ °K:

$$\Delta W \simeq \varkappa T_{\text{equiv}} (\hat{\tau}/\tau^*)$$

(see expression (1.13)). Substituting here $\tau^* = 2{,}400$ sec and $\hat{\tau} = 23$ sec, we obtain $\Delta W = \kappa 46$ °K $= 6.5 \cdot 10^{-15}$ erg. We note that since in the estimates we used the half-width of the confidence interval $\Delta\phi_{con}$, in (1.13) a numerical factor was dropped. Thus, the resulting estimates of ΔW correspond to a low reliability level (about 0.7). /29

We will cite two other figures for characterizing the described apparatus. The value $\Delta\phi_{con} = 1.8 \cdot 10^{-6}$ rad corresponds to a threshold amplitude of the sinusoidal resonance train with a duration $\hat{\tau} = 23$ sec, equal to $F_0 = 1 \cdot 10^{-9}$ dyne (if the force was applied to the edge of the pendulum). Such a force can be created by the pressure of a light flux with the intensity $N \simeq 20$ erg/sec.

§ 2. Dynamic Effect of Instrument Registering Small Oscillations on a Mechanical Oscillator.

In the preceding section we examined the case of detection of the minimum effect on a mechanical oscillator with a large time constant. The instrument registering the oscillator displacement was considered ideal, i.e., it was assumed that it exerts no effect on the oscillator.

In this section we will analyze the dynamic effects caused by different small displacement indicators (detectors). In other words, we will examine the effect of an indicator on the characteristic frequency and damping of a mechanical oscillator. In striving to achieve the smallest possible response, these effects must be taken into account, and as will be seen below, in some cases they must be carefully compensated.

Electronic small displacement indicator. We will examine the effect of a capacitive transducer on a mechanical oscillator. The capacitive transducer is one of the most sensitive electronic devices used in registering small mechanical displacements. The capacitive transducer is usually an electric circuit with an air capacitor, one of whose plates is movable (Figure 5). A displacement of the oscillator mass results in a change in the capacitor gap d, and accordingly, a retuning of the characteristic frequency of the electric circuit. If the frequency of the electric generator Ω_{gen} is displaced approx- /30
imately by the half-width of the electric circuit band (Figure 6), a change Δd in the capacitor gap leads to the maximum change ΔU in the amplitude of the electric voltage $U(\tau, d)$ which can be registered by an amplitude voltmeter. The ΔU quantity is

$$\Delta U \simeq 0.5 U_0 Q_{el} \frac{\Delta d}{d}, \qquad (2.1)$$

where Q_{el} is the electric circuit quality, U_0 is the amplitude of the electric voltage across the capacitor. Usually Ω_{gen} and the circuit resonance frequency Ω_{opt} are substantially greater than ω_{mech}.

Capacitive transducers can be used in resolving amplitudes of mechanical displacements up to 10^{-12} cm and quasistatic displacements up to 10^{-9} cm (for further details, see the Appendix).

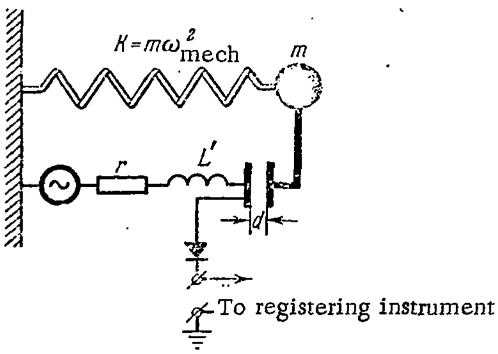

Figure 5

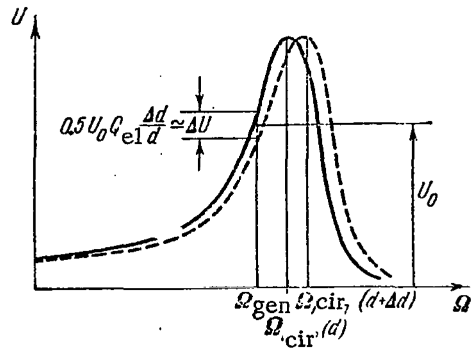

Figure 6

The mechanism of the influence of the capacitive transducer on the dynamic characteristics of a mechanical oscillator can be qualitatively represented as follows. The capacitor plates are attracted with the force /31

$$F(\tau,\ d) = -\frac{SU^2(\tau,\ d)}{8\pi d^2(\tau)}, \qquad (2.2)$$

where S is the area of the capacitor plates, and U is the potential difference.

Since $\Omega_{gen} >> \omega_{mech}$ (usually 6 or 7 orders of magnitude), for a mechanical oscillator only slow Fourier components $\overline{F\ (\tau,\ d)}$ are important. We note that the greater the sensitivity of the sensor, the greater will be the U/d value, and accordingly, the greater will be the mean force of attraction. The $\overline{F\ (\tau,\ d)}$ value is strongly dependent on the instantaneous position of the mass m, since displacements of mass lead to a retuning of the circuit and a change in $U\ (\tau,\ d)$. This means that the additional differential rigidity $\Delta K = \partial\overline{F\ (\tau,\ d)}/\partial d$ is added to the mechanical oscillator due to the $F\ (\tau,\ d)$ force; this changes the characteristic frequency of oscillator small oscillations ω_{mech} [11]. This rigidity is introduced with a lag approximately equal to the time for stabilization of electric oscillations in the circuit. The lag for a reactive element, as is well known, leads to regeneration or degeneration in an oscillatory system. Thus, a capacitive transducer can also change the oscillator relaxation time.

Quantitative estimates for these two effects are easily obtained. The amplitude of the electric voltage $U_\sim$ across the transducer circuit is

$$U_\sim = U_0\left[(1-\gamma^2)^2 + \frac{\gamma^2}{Q^2_{\text{el}}}\right]^{1/2}, \tag{2.3}$$

where U_0 is the amplitude of oscillator oscillations, $\gamma\ (d) = \Omega_{gen}/\Omega_{cir}$.

Assume that the tuning is such that $\gamma_0 = 1 + \beta/2Q_{el}$, and that β is about unity. When $\beta = \pm 1$ and greater than Q_{el} the amplitude $\overline{U}_\sim \simeq Q_{el}U_0\ (2)^{-1/2}$, and the transducer response is close to the maximum. The $\overline{F\ (d)}$ value for the case of small deviations of x from d_0, corresponding to γ_0, is equal to

$$\overline{F(x)} = \frac{U_0^2 Q^2_{\text{el}} S}{16\pi d_0^2}\left[1 \pm \left(\frac{2|\beta|}{1+\beta^2} Q_{\text{el}} \frac{x}{d_0}\right) + \ldots\right]. \tag{2.4}$$

In (2.4) we have omitted the terms $(Q_{el}\ \frac{x}{d_0})^2$, $(Q_{el}\ \frac{x}{d_0})^3$, etc. A positive /32
sign in (2.4) corresponds to the right slope of the resonance curve; a negative sign corresponds to the left slope (the signs are the same as for β).

From (2.4) we obtain the differential mechanical rigidity ΔK:

$$[\Delta K]_{\text{max}} = \left[\frac{\partial \overline{F\ (x)}}{\partial x}\right]_{\text{max}} = \pm \frac{U_0^2 Q^3_{\text{el}} S}{1\ \pi d_0^3}\ . \tag{2.5}$$

Thus, on the left slope of the resonance curve the transducer introduces a negative differential rigidity into the mechanical oscillatory system, i.e., it increases the period of oscillations; on the right slope it introduces a positive differential rigidity.

If it is taken into account that the ΔK value is introduced with the lag $\tilde{\tau} \simeq Q_{el}/\Omega_{cir}$, the equation for oscillator small oscillations assumes the following form:

$$m\ddot{x} + H_{\text{mech}}\dot{x} + (K \pm \Delta K_{\tilde{\tau}})x = 0, \tag{2.6}$$

where H_{mech} is the friction coefficient in the oscillator. A plus sign corresponds to the right slope, whereas a minus sign corresponds to the left slope of the resonance curve for an electric circuit on which the frequency Ω_{gen} is tuned. It is easy to see that the lag in positive rigidity leads to regeneration, whereas lag in negative rigidity leads to degeneration. The self-excitation condition for the right slope has the very simple form:

$$H_{\text{mech}} \cong \Delta K \tilde{\tau}. \tag{2.7}$$

Hence, by substituting (2.5) and the expression for $\tilde{\tau}$, it is possible to compute the minimum voltage for the circuit $U_{\sim}^{*}$ at which oscillatory instability arises in the oscillator [11]

$$U_{\sim}^{*} = \sqrt{\frac{8\pi\omega_{\text{mech}} m d_0^3}{Q_{\text{mech}} S \tilde{\tau} Q_{\text{el}}}} \tag{2.8}$$

where Q_{mech} is the oscillator quality when $U_{\sim}^{*} \to 0$. As can be seen from (2.8), the greater the time constant τ^* and Q_{mech}, the lesser will be the $U_{\sim}^{*}$ values at which oscillatory instability arises. /33

Below, as an illustration, we give the results of measurements of the dynamic parameters of a torsional pendulum, whose small oscillations were registered by a capacitive transducer (for further details, see [11]). The pendulum had the following data: $Q_{mech} \simeq 4 \cdot 10^3$, $m = 4$ g, $d_0 = 0.1$ cm, $S = 4$ cm^2, $\omega_{mech} \simeq 2\pi \cdot 0.1$ sec^{-1}. The capacitive transducer circuit had $\Omega_{cir} = 2\pi \cdot 6 \cdot 10^6$ sec^{-1}, $Q_{el} = 50$.

Table 2 gives the period of oscillations τ_0 and the decrement (increment) ϑ of the amplitudes of pendulum oscillations for different amplitudes of electric voltage $U_{\sim}$ in the circuit with tuning of the generator for the left and right slopes of the resonance curve.

The change in the period τ_0 agrees well with expression (2.5).

The table shows that oscillatory instability for a torsional pendulum appeared when $U_{\sim}^{*} \simeq 3$ V. This is somewhat less than follows from the estimate which can be made using (2.8). The possible reason for this disagreement is that the self-excited oscillator used in the experiment was quite heavily loaded by the transducer circuit and "responded" to a change in its parameters with some additional $\tilde{\tau}$ (for further details see [11]). /34

TABLE 2

Left Slope				
U, V	1	3	5	7
τ_0, sec	11.0	11.4	12.2	13.0
ϑ	$+ 2.4\cdot10^{-3}$	$+ 4.2\cdot10^{-3}$	$+ 1.0\cdot10^{-2}$	$+ 2.4\cdot10^{-2}$
Right Slope				
U, V	1	3	5	7
τ_0, sec	11.0	10.9	10.8	10.6
ϑ	$+ 1\cdot10^{-4}$	$- 1.4\cdot10^{-3}$	$- 3.3\cdot10^{-3}$	$- 2.1\cdot10^{-2}$

As can be seen from the above, the two oscillatory systems, a mechanical oscillator and an electric circuit, having substantially different frequencies of characteristic oscillations (in the described example $\Omega_{cir}/\omega_{mech} \simeq 6\cdot10^7$), are related to Coulomb interaction so that oscillatory instability can arise when there is quite small friction in the mechanical oscillator. Such a relationship is not only manifested in precise experiments with test bodies in which relatively low-power self-excited oscillators are used as sensors. This effect was also significant in powerful accelerators in which there was an oscillatory mechanical instability of the diaphragms forming part of the electric resonators [12].

In experiments with test bodies, as pointed out in § 1, in order to increase the response it is desirable to decrease H_{mech}, relating the test mass m with the laboratory. On the other hand, when measuring small oscillations when using electronic transducers, it is necessary to increase $U_\sim$ and decrease d_0 (see formula (2.1)), which leads to the appearance of an additional $H \simeq \Delta K\tilde{\tau}$, which either increases the dissipation or leads to oscillatory instability.

Thus, these two requirements are contradictory. Accordingly, the use of electronic transducers for registering small displacements is evidently feasible only in experiments with relatively large masses (and accordingly, large H_{mech}).

If it is possible to use two transducers arranged symmetrically on a test mass, the examined effects can be considerably compensated. Another obvious recommendation is to decrease the $\tilde{\tau}$ value, i.e., increase Ω_{cir}. However, as will be demonstrated somewhat below, optical systems for indicating small oscillations also exert an influence on the dynamic parameters of mechanical oscillators, and as in the case of electronic transducers, can lead to the appearance of oscillatory instability.

Optical indicators of small displacements. Two principal optical methods /35 in different modifications are known for indicating small mechanical displacements (for further details see the Appendix). In the first method (this is sometimes called the "knife and slit" method or the "optical lever" [13]), the optical image of one diffraction grating, obtained using an objective, is matched with a second grating which usually has the same interval. The displacement of one of these gratings parallel to the other causes a light flux modulation. Using this method it is possible to register quasistatic displacements of about 10^{-12} cm [14].

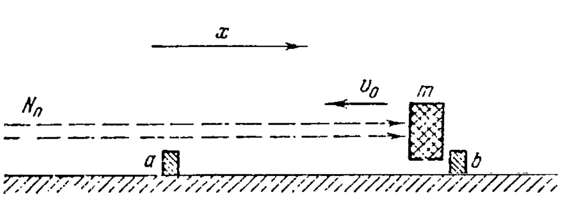

Figure 7

The second optical method for measuring small displacements is similar to the capacitive transducer examined in the preceding segment of this section. The mirrors of a Fabry-Perot resonator are tuned in such a way that the frequency of the monochromatic source falls on the slope of a resonance curve of the fundamental type of resonator oscillations. Small displacements of the mirrors in the direction of the resonator axis lead to an intense modulation of the light flux passing through it. Using this method it has been possible to register quasistatic displacements of about 10^{-13} cm [15].

Now we will examine the dynamic effect of such optical indicators on a mechanical oscillator. First we will estimate the dissipation introduced by a homogeneous light flux in a mechanical and harmonic oscillator whose mass experiences translational motions along the x-axis (Figure 7) [16].

The anharmonic oscillator consists of a mass experiencing oscillatory motion in the x direction between two absolutely elastic supports a and b. Thus, in the absence of dissipation the velocity during one half-period is constant and equal to $+ v_0$; in the course of the other half-period it is equal to $- v_0$. We will assume that a light flux N_0, completely absorbed by the mass m, is directed along the x axis. It is clear that with movement from b to a the mass m must receive a greater field impulse than during movement from a to b. Simple computations show that a decrease in the velocity Δv of the mass m during a full period is equal to /36

$$\Delta v \simeq \frac{N_0 \tau_0}{cm} \cdot \frac{2v_0}{c}, \tag{2.9}$$

where τ_0 is the full period of oscillations. Such a velocity decrease corresponds to the friction coefficient

$$H_{em} = \frac{2N_0}{c^2}. \tag{2.10}$$

Here H_{em} is the coefficient of friction of electromagnetic origin. Obviously, for a harmonic oscillator H_{em} differs from (2.10) by a factor of the order of unity.

The H_{em} parameter is small, even for relatively powerful fluxes. As a comparison we point out that the friction caused by a rarified gas H_{gas} has the same order of magnitude as H_{em} only in a deep vacuum. For a sphere with the radius a

$$H_{gas} \simeq 4a^2 \mu^{1/2} (\varkappa T)^{1/2} \tilde{f}, \tag{2.11}$$

where $\tilde{f}$ is the concentration of molecules of a gas whose mass is μ and whose temperature is T. In a hydrogen atmosphere with $p = 10^{-11}$ mm Hg, $T = 100°K$, $a = 1$ cm, the coefficient $H_{gas} \simeq 3 \cdot 10^{-13}$ g/sec. If it is assumed that $N_0 = 10^8$ erg/sec, then $H_{em} = 2 \cdot 10^{-13}$ g/sec.

In a case when the mass m in the example considered above (Figure 7) reflects well the radiation, the effect is retained and will be twice as great: $H_{em} = 4N_0/c^2$.

If the test body moves in a direction perpendicular to the light flux (as occurs in the case of an optical lever), the flux will also introduce a small additional friction caused by the Robertson-Poynting effect [17, 18]:

$$H_{em} = (1 - R)\frac{N_0}{c^2}, \tag{2.12}$$

where R is the reflection coefficient, N_0 is part of the light flux intensity incident on the body[1]. /37

Thus, the use of any type of light flux modulators in experiments with test bodies leads to the appearance of a relatively small friction $H_{em} \sim N_0/c^2$, comparable only with the friction in gases highly rarified for terrestrial conditions. The time constant $\tau^* \simeq mc^2/N_0$, corresponding to the friction coefficient H_{em}, is extremely large. For example, if $N_0 = 10^3$ erg/sec, $m = 10$ g, then $\tau^* \simeq 10^{18}$ sec (!). This indicates, if we take into account the expressions derived in the preceding section (for example, expression (1.15)), the presence of an enormous response reserve in experiments with test bodies. In the laboratory at the present time it has only been possible to obtain values $\tau^* \simeq 10^{+9}$ sec, and such time constants have not been used in obtaining maximum response. We note that the use of a time constant $\tau^* \simeq mc^2/N_0$ cannot be used in expressions for minimum detectable forces in the case of arbitrary N_0 values. A more detailed analysis of the attainable resolution in experiments with test bodies for the case when the

[1] The Robertson-Poynting effect, like the light friction effect in the oscillatory motion of an oscillator which reflects light, has still not been discovered experimentally.

only source of fluctuations is the measuring instrument will be given in section 3.

The light flux in which the text body is situated, in addition to a relatively small friction, also introduces a relatively large differential rigidity

$$K_{\text{light}} \simeq \frac{1}{c} \frac{\partial N(r)}{\partial r}$$

(it can be arbitrarily called "light rigidity"). In the case of a torsional oscillator, it is possible to determine the "light rigidity" in a homogeneous flux ($\partial N(r)/\partial r = 0$) as well, as can be seen from Figure 8. If the oscillator is a symmetrical dumbbell with plates on the ends, the light flux in the directions aa' create a negative "light rigidity", whereas in the directions bb' they create a positive "light rigidity", equal to /38

$$K_{\text{light}} \simeq \pm 2 \frac{\Phi}{c} (1 + R) Sl \sin 2\alpha_0, \qquad (2.13)$$

where Φ is the light flux density, S is the area of the plates, R is the reflection coefficient, α_0 is the angle between the direction of the light flux and the dumbbell. The rigidity sign is dependent on the direction of the light fluxes. When $\alpha_0 = \pi/4$, $l = 10$ cm, $R \cong 1$, $S = 1$ cm^2, $\Phi = 10^7$ erg/sec$\cdot$cm^2 and the "light rigidity" $K_{light} \simeq 1.3 \cdot 10^{-2}$ dyne$\cdot$cm.

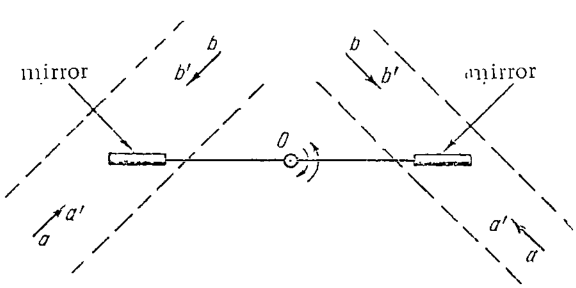

Figure 8

As an illustration, we give the parameters of a torsional pendulum whose period of characteristic oscillations is retuned due to "light rigidity." The pendulum is a dumbbell 14 cm in length with light aluminum lugs on the ends. The moment of inertia of the dumbbell was 0.2 g$\cdot$cm^2. The dumbbell was suspended horizontally in an evacuated flask on a tungsten filament 8 μ in diameter and 20 cm in length. In the absence of light fluxes the period of characteristic oscillations for the dumbbell was 15.8 sec. If the light fluxes are directed as shown in Figure 8, with a flux density $\Phi \simeq 2 \cdot 10^6$ erg/sec·cm^2 the period of oscillations changed from 13.9 sec (direction of fluxes *aa'*) to 19.7 sec (direction of fluxes *bb'*). By smoothly changing the flux density or the angle of incidence of the flux on the plates, it is possible to obtain a smooth retuning of the period of characteristic pendulum oscillations.

The use of Fabry-Perot resonators (or other interferometers) in theory makes it possible to obtain a greater response in registering small displacements than the use of optical levers (see Appendix). However, an increase in response results in an increase in the dynamic influence of such an indicator on the mechanical oscillator. We will estimate these effects in the experimental model shown in Figure 9. Assume that one of the mirrors in the Fabry-Perot resonator is in a fixed position, whereas the second is displaced together with the mass *m* of the oscillator in the direction of the resonator axis. In order to obtain the maximum change in photodetector current with displacement of the mass it is necessary to tune the resonator in such a way that the frequency of the optical source falls in the middle of the resonance curve slope. This will result in maximum response, but at the same time the light pressure on the mirror will greatly change during the displacement. The "light rigidity" arising in this way will be relatively great. Its maximum value $[K_{light}]_{max}$ is equal to /39

$$[K_{\mathrm{light}}]_{\mathrm{max}} \simeq \pm \frac{N_0}{c} \frac{4\pi}{\lambda_0 (1 - R)^2}, \tag{2.14}$$

where N_0 is source intensity, λ_0 is the resonance wave length, and R is the mirror reflection coefficient. The sign of K_{light} is dependent on which of /40
the resonance curve slopes is used for source tuning. It is generally easy to compute the K_{light} value. This is done using the well-known equations for a Fabry-Perot resonator [19] and computing the light pressure intensity on the mirrors as a function of the distance between them.

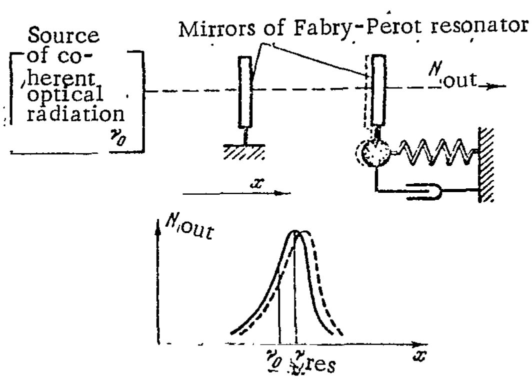

Figure 9

If in (2.14) it is assumed that N_0 = 300 MW, $\lambda_0 = 6 \cdot 10^{-5}$ cm, $(1 - R) = 1 \cdot 10^{-2}$, then $[K_{light}]_{max} \cong \pm\ 2 \cdot 10^{+5}$ dyne/cm.

As in a capacitive transducer, the rigidity K_{light} in the Fabry-Perot resonator is not introduced instantaneously but with some lag $\tilde{\tau} \simeq l/c\,(1 - R)$. Accordingly, in addition to the rigidity in an oscillator with a Fabry-Perot resonator there will be friction, whose sign is determined by the rigidity sign

$$[H_{em}]_{max} \cong \pm \frac{N_0 4\pi l}{c^2 \lambda_0 (1 - R)^3}. \tag{2.15}$$

The K_{light} maximum was used in expression (2.15). Thus, in a Fabry-Perot resonator there is additional friction $4\pi l/\lambda_0 (1 - R)^3$ times greater than the friction introduced into the oscillator by "free" light fluxes (compare the expressions (2.15), (2.10) and (2.12)).

If $|-H_{em}| > H_{mech}$, the oscillator oscillations will become increasingly greater until the amplitude of the oscillations, as a result of the nonlinear dependence K_{light} (l), becomes stationary. This effect is similar to the already described interaction between two oscillatory systems (radio-frequency and mechanical) in the case of a capacitive transducer. In a capacitive transducer the interaction caused by the Coulomb attraction of the plates is nonlinear; in the Fabry-Perot resonator the light pressure is also quadratically dependent on field amplitude. As in the case of a capacitive transducer, the K_{light} and H_{em} introduced by the Fabry-Perot resonator can be compensated.

In concluding this section we will give a numerical estimate. If $l = 10^2$ cm, $\lambda_0 = 6\cdot10^{-5}$ cm, $(1 - R) = 1\cdot10^{-2}$, $N_0 = 300$ mW, then $[H_{em}]_{max} = \pm 6\cdot10^{-2}$ g/sec. When $m = 10$ g, $\omega_{mech} = 2\pi\cdot10^2$ sec^{-1} and $Q_{mech} = 10^5$, we have: $[H_{em}]_{max} = H_{mech}$.

Radiometric oscillatory instability. The radiometric effect, as in the case of light pressure, can create an additional differential rigidity. With /41 a vacuum of about 10^{-4} mm Hg the radiometric pressure is already an order of magnitude less than the light pressure and accordingly the "radiometric rigidity" is an order of magnitude less. However, the thermal inertia of the mass of a mechanical oscillator can be relatively great (several seconds or more). This means that the radiometric rigidity is introduced with a lag. Thus, even a radiometric rigidity of small magnitude can introduce into a mechanical oscillatory system a positive or negative (depending on the rigidity sign) friction. The sign for "radiometric rigidity" is the same as for "light rigidity" (this effect was discovered by the author in collaboration with V. N. Rudenko).

Table 3 gives data on the relaxation time τ^* of mechanical torsional oscillators used in checking the existence of "light rigidity" (see Figure 8). The τ^*_0 value is the relaxation time without light fluxes; τ^*_- is the pendulum relaxation time when the direction of the light fluxes is along aa', τ^*_+ is the pendulum relaxation time when the light fluxes are directed along bb' (see Figure 8). The left column gives the pressure in the flask in which the pendulum was placed.

TABLE 3

p, mm Hg	τ^*_0, min	τ^*_-, min	τ^*_+, min
$5\cdot10^{-7}$	16.0	10.0	26.0
$2\cdot10^{-8}$	17.5	15.0	18.5

The table shows that the difference $|\tau^*_0 - \tau^*_-|$ and $|\tau^*_0 - \tau^*_+|$ increases with a deterioration in the vacuum. At higher pressures an oscillatory instability appears for this pendulum.

In summarizing the considered effects of the dynamic influence of electronic and optical instruments on mechanical oscillators, we must once again emphasize the theoretical possibility of compensating such a dynamic effect; however, this will not change the fluctuation effect of these instruments on oscillators.

§ 3. Classical and Quantum Fluctuation Effect of a Measuring Instrument on a Mechanical Oscillator /42

In § 1 in this chapter, we examined methods for detecting weak regular effects on a mechanical oscillator having a large time constant τ^*. It was assumed that the instrument registering the change in movement of the mass m of the oscillator, caused by a regular effect, was ideal. In other words, the assumption was made that the instrument exerts neither dynamic nor fluctuation effects on the mechanical oscillators. In § 2 we examined the dynamic effect exerted on an oscillator by different types of indicators, an effect which can be compensated.

An increase in the time constant τ^* of an oscillator, or its equivalent, a decrease in the friction coefficient H, being a source of a fluctuating force, leads to an increase in the resolution in detecting the effect of regular $F(\tau)$ forces on an oscillator. It is clear that in the case of sufficiently small H (sufficiently large τ^*) a fluctuation effect begins to appear from the small displacements indicator. It is important to note that these fluctuations (of nontechnical origin) cannot be compensated.

With a further increase in τ^* (decrease in H) the minimum detectable $F(\tau)$ values will be determined exclusively by the fluctuation effects from the indicator. In other words, in macroscopic experiments with test bodies with sufficiently large τ^* the situation is similar to that in quantum mechanics: it is impossible to exclude the measuring instrument from consideration.

We will examine the fluctuation effect of two types of instruments for registering small oscillations of macroscopic mechanical oscillators. We will first discuss the electronic detector of small displacements.

Figure 10 shows a diagram of an experiment in which a capacitive transducer was used as the small displacement indicator. The generator frequency Ω_{gen} is slightly (by approximately half the width of the electric circuit resonance curve) displaced relative to the circuit resonance frequency Ω_{cir}. /43 Small oscillations of the mass m (and accordingly, the capacitance C) lead to large changes in amplitude of the circuit electric voltage; these are registered after rectification by the measuring instrument.

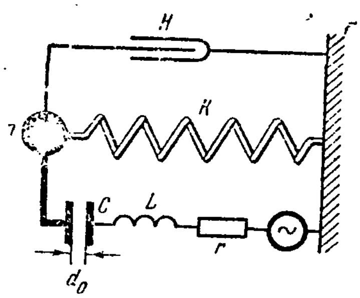

Figure 10

We will assume that by symmetrical arrangement of the two (or more) capacitor plates it was possible to compensate completely the dynamic effect of such a sensor on the mechanical oscillator (see § 2). We will also assume that fluctuations in frequency and amplitude of the self-excited oscillator forming part of the sensor (see Appendix) have been eliminated or strongly compensated and that the H value and accordingly the mechanical fluctuations caused by H can be neglected (see formula (1.15)). Then the thermal fluctuations of electric voltage across the resistor r in the electric circuit lead to capacitance fluctuations. The Coulomb interaction of the capacitor plates, whose charge fluctuates, will be the only force F^*_{fl}, against whose background it is necessary to detect the regular F (τ) force.

Since the Coulomb attraction of the capacitor plates is dependent on the square of the voltage between them, F^*_{fl} will also be dependent on the amplitude of the electric voltage $U_\sim$ caused by the self-excited oscillator. Simple computations give the following expression for the spectral density of F^*_{fl} under the condition that $\omega_{mech} << \Omega_{cir}$ and $\Omega_{gen} = \Omega_{cir}$ $(1 \pm 1/2Q_{el})$:

$$(F^*_{fl})^2_\omega \simeq \frac{S^2 U^2_\sim Q^2_{el} \varkappa T r}{2\pi^2 d_0^4}, \qquad (3.1)$$

where S is the area of the capacitor plates, $U_\sim$ is the amplitude of the circuit electric voltage, Q_{el} is its quality, r is the active resistance in /44 the circuit, whose temperature is T, d_0 is the mean distance between the plates.

In order to detect $F(\tau)$ it is necessary that

$$[F(\tau)]_{min} \geqslant \zeta \sqrt{(F_{fl})^2 \Delta f}, \tag{3.2}$$

where Δf is the frequency band within which most of the $F(\tau)$ spectrum lies; ζ is a factor having the order of several units and dependent on the selected level of detection reliability (if only a statistical error of the first kind is stipulated, ζ coincides with the $t_{1-\alpha}$ quantile of Student's t).

As can be seen from (3.1) and (3.2) in order to decrease the minimum detectable $[F(\tau)]_{min}$ value it is necessary to decrease the amplitude of the electric oscillations in the circuit $U_{\sim}$, decrease S and Q_{el}, and increase the gap d_0. However, this requirement leads to a decrease in response in measuring the $[F(\tau)]_{min}$ value. If $\omega_{mech} \ll \Omega_{cir}$ and $\Omega_{gen} = \Omega_{cir}(1 \pm 1/2Q_{el})$, the minimum detectable displacements $[x(\tau)]_{min}$ when using a capacitive transducer will be

$$[x(\tau)]_{min} = \zeta \frac{4d_0}{U_{\sim}} \sqrt{\varkappa T r \Delta f}. \tag{3.3}$$

Thus, $[F(\tau)]_{min}$ should cause a displacement greater than $[x(\tau)]_{min}$, and therefore, $U_{\sim}/d_0$ should be quite large; on the other hand, condition (3.2) should be satisfied; it therefore follows that $U_{\sim}/d_0$ should not be greater than a certain value. The existence of these two such contradictory requirements is indicative of the existence of an optimum strategy: the $[F(\tau)]_{min}$ value must simultaneously satisfy condition (3.2) and cause a displacement greater than (3.3). This means that there is an optimum value $[U_{\sim}]_{optim}$, which can be computed having only preliminary information on the $F(\tau)$ form. The minimum detectable force $[F(\tau)]_{min}$ actually corresponds to this $[U_{\sim}]_{optim}$.

We will find $[F(\tau)]_{min}$ for the case $F(\tau) = F_0 \sin \omega_{mech} \tau$ during the time interval $0 \leqslant \tau \leqslant \hat{\tau}$ and $F(\tau) = 0$ outside this time interval. Then /45

$$\Delta x = F_0 \hat{\tau} (2m\omega_{mech})^{-1}. \tag{3.4}$$

Substituting into (3.4) an expression for $[x\ (\tau)]_{min}$, in which $\Delta f = 1/\hat{\tau}$, and solving this equation jointly with (3.2) for F_0 and $U_\sim$, we obtain

$$[F_0]_{min} = \zeta \frac{4}{\hat{\tau}} \sqrt{\pi \sqrt{2} \frac{\varkappa T m \omega_{mech}}{\Omega_{cir}}} \tag{3.5}$$

$$[U_\sim]^2_{optim} = \frac{8\pi^2 \sqrt{2}\omega_{mech} m d_0^3}{\tau S Q_{el}}. \tag{3.6}$$

The expressions for $[\hat{F}]_{min}$ and $[U_\sim]_{optim}$ in the case $F\ (\tau) = \hat{F}$ during the interval $0 \leqslant \tau \leqslant \tilde{\tau}$, if $\hat{\tau} << 1/\omega_{mech}$, will differ from (3.5) and (3.6) only in the coefficient 2, since for a short impulse $\Delta x = \hat{F}\hat{\tau}\ (m\omega_{mech})^{-1}$.

These simple computations reveal that in the case of a sufficiently small friction in a mechanical oscillator, the equilibrium thermal fluctuation in the electronic instrument registering the small oscillations will determine the minimum detectable regular force $[F\ (\tau)]_{min}$, provided that the detector is tuned in the best way, taking into account preliminary information on the $F\ (\tau)$ form. As can be seen from (3.5), the expression for $[F_0]_{min}$ includes only the temperature T and the characteristic frequency of circuit oscillations Ω_{cir}, whereas the other parameters of the electronic device are determined only by $[U_\sim]_{optim}$. It is also extremely important that with optimum tuning $[F_0]_{min}$ decreases as $(\hat{\tau})^{-1}$. However, if the fluctuations caused by H are decisive, the $[F_0]_{min}$ value decreases as $(\hat{\tau})^{-1/2}$ (see formula (1.15)).

It follows from (3.5) that in order to reduce the threshold $[F_0]_{min}$ it is advantageous to decrease the $\omega_{mech}/\Omega_{cir}$ ratio. Since in deriving (3.5) and (3.6) we used the classical Nyquist theorem, not the Kallen-Welton expressions (for example, see [20]), formulas (3.5) and (3.6) lose their validity when $\kappa T/\Omega_{cir} \simeq \hbar$. The role of quantum fluctuations is more conventiently examined in the case of an optical indicator.

The computation of $[F\ (\tau)]_{min}$ given above for two specific cases of the $F\ (\tau)$ form is easily repeated for any other $F\ (\tau)$ form. /46

In (3.5) we will assume that $\kappa T/\Omega_{cir} = 10\ \hbar$, $\omega_{mech} = 1\ \mathrm{sec}^{-1}$, $m = 1$ g, $\hat{\tau} = 10^3$ sec, $\zeta = 2$. Then $[F_0]_{min} \simeq 1.6 \cdot 10^{-15}$ dyne. With the same ω_{mech}, m and $\hat{\tau}$, this $[F_0]_{min}$ value is 7-8 orders of magnitude less than it is possible to resolve in present-day laboratory experiments. The next chapter (§§ 5, 6, 7) will give a more detailed comparison of the attained sensitivity in experiments with test bodies and the theoretically attainable response.

Now we will examine the fluctuation effect of an optical indicator on a macroscopic oscillator. Such an effect is caused by light pressure fluctuations and with elimination of technical instabilities of light forces it has a quantum nature. Before proceeding to an analysis of this problem, we will discuss two (nonclassical) results for a mechanical oscillator. There is a precise solution [21] (I. I. Gol'dman, V. D. Krivchenkov) for the probability p_{0n} of transition of a quantum oscillator from the fundamental to the n-th state after a time-finite exposure to a classical force $F(\tau)$:

$$p_{0,n} = e^{-y} \frac{y^n}{n!}, \tag{3.7}$$

$$y = \frac{1}{2\hbar\omega_{\mathrm{mech}} m} \left| \int_{-\infty}^{+\infty} F(\tau)\, e^{-i\omega_{\mathrm{mech}}\tau} d\tau \right|^2, \tag{3.8}$$

where m is the oscillator mass, ω_{mech} is its characteristic frequency. If $F(\tau)$ has the form of a train of sinusoidal oscillations with the amplitude F_0, a frequency coinciding with ω_{mech}, and a duration $\hat{\tau}$, then

$$y = F_0^2 \hat{\tau}^2 (2\hbar\omega_{\mathrm{mech}} m)^{-1}. \tag{3.9}$$

A force with the amplitude F_0 can be considered detected if $\sum_{i=1}^{\infty} p_{0i} = (1 - \alpha)$ is quite close to 1. The α parameter, as in §, has the sense of a statistical error of the first kind. Expression (3.9) can be rewritten as follows: /47

$$(F_0)_{1-\alpha} = \frac{\sqrt{y}}{\hat{\tau}} \sqrt{2\hbar\omega_{\mathrm{mech}} m}. \tag{3.10}$$

Here $\hat{\tau}$ is the time of exposure to the $F(\tau)$ force, but not the time expended on measurement τ_{meas}. When $y = 2, 3, 4$, the $(1 - \alpha)$ value is equal to 0.86, 0.95, 0.98 respectively.

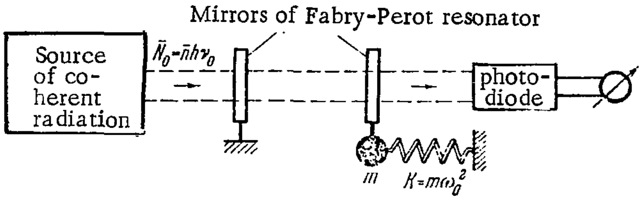

Figure 11

Expression (3.10), like (3.5) (if in the latter $\kappa T/\Omega_{cir}$ is replaced by $\hbar$), does not, however contain an answer to the question of the minimum detectable amplitude of the force $[F_0]_{min}$ for a stipulated time τ_{meas}. It is possible to "learn" whether an oscillator has responded to the $F(\tau)$ force, i.e., whether it has "absorbed" one or more quanta only by awaiting spontaneous radiation, whose time is not always easy to determine[1]. Thus, it is necessary to supplement the oscillator by an instrument in which there are also quantum fluctuations, and take into account its inverse effect on the oscillator.

Before proceeding to an examination of this problem, we note that the probability of transition of the oscillator under the influence of the same force from the n-th state into the next state will be the greater, the greater the n value. Using well-known expressions, derived by the methods of perturbation theory for p_{nm} [33], it is easy to find that if $F(\tau)$ has the form of a train with the duration $\hat{\tau}$, the amplitude F_0, and a frequency coinciding with ω_{mech}, the probability of transition from the n-th to the $(n + 1)$-st state is /48
close to unity if[2]

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[1] For example, if the rigidity in the oscillator was created by a gravity field gradient (as was the case, for example, in some experiments made by Eotvos [22]), in order to determine the time of spontaneous radiation it is necessary to quantize the gravitational interaction.

[2] The probability

$$p_{nm} = \frac{e^{-y} y^{n-m} [Q_m^{n-m}(y)]^2}{n!m!}, \quad \text{where} \quad Q_s^l(y) = \sum_{\mu=0}^{s} \frac{(-1)^{\mu} y^{s-\mu} s!(s+l)!}{(s-\mu)!(s+l-\mu)!\mu!},$$

and y coincides with expression (3.8). Assuming y to be small (if for no other reason than to insure that p_{nn+1} will be close to 1), and also assuming that $F_0(\tau) = F_0 \sin\omega_{mech}\tau$ during the interval τ, we obtain the approximate expression (3.11).

$$F_0 \simeq \frac{1}{\hat{\tau}} \sqrt{\frac{2\hbar\omega_{mech}\, m}{n}} \qquad (3.11)$$

Thus, the greater the n value the lesser will be the detectable force (with the same reservations on the observation time as for expression (3.10)). Expression (3.11) shows that the greater the initial amplitude of oscillator oscillations, the lesser will be the role played by the discreteness of its energy levels and the more important will be the fluctuations in the instrument registering these oscillations. Accordingly, in the approximate problem examined below we will assume the oscillator to be classical and take into account only the quantum fluctuations in the optical instrument.

We will assume that a Fabry-Perot resonator (Figure 11) was used as the instrument registering the small oscillations of the classical oscillator. One of the resonator mirrors, a source of coherent optical radiation with the intensity N_0 and frequency ν_0, attached to the oscillator mass, excites in the resonator oscillations in the fundamental type of oscillations. Motion of the mirror attached to the mass m results in a modulation of the light flux emerging from the resonator and this flux is registered by a quantum counter.

In order to obtain the maximum response such a resonator must be slightly (by $\Delta\nu = \nu_0/2Q_{res}$, where Q_{res} is the resonator quality) detuned relative to the frequency ν_0. With such tuning, as in the case of a capacitive transducer, the mechanical oscillations of the mirrors give the maximum light flux modu- /49
lation intensity (see Appendix).

We will assume that the use of a compensated measurement circuit can eliminate the rigidity ΔK and the friction H_{em} introduced in this experiment by the optical indicator and associated with the high resonator quality. With respect to the light source, we will assume that technical instabilities of intensity and frequency have been compensated and that the spectral density of source frequency deviation w (ν) and the spectral density of the light flux modulation M_ω^2 are equal to

where

$$P_{nm} = \frac{e^{-y} y^{n-m} [Q_m^{n-m}(y)]^2}{n!m!},$$

$$Q_s^l(y) = \sum_{\mu=0}^{s} \frac{(-1)^{s-\mu} y^{s-\mu} s!\,(s+l)!}{(s-\mu)!\,(s+l-\mu)!\,\mu!},$$

$$w(\nu) = A \frac{8\pi h\nu_0 (\Delta\nu_{res})^2}{N_0}, \tag{3.12}$$

$$M_\omega^2 = A' \frac{2h\nu_0}{N_0}, \tag{3.13}$$

where ν_0 is the mean source radiation frequency, N_0 is the mean intensity at the resonator input, $\Delta\nu_{res}$ is the width of the resonator frequency band, A and A' are dimensionless factors. If the photons are emitted independently and the frequency ν_0 is sufficiently great so that there are no characteristic Bose fluctuations, then $A' \simeq 1$. In modern gas lasers $A' \simeq 10^2$ for the low modulation frequencies and with an increase in modulation frequency $A' \simeq 1$ (for example, see [24]). Approximately the same situation exists for the factor A (see [25]). In subsequent computations it will be assumed that $A = A'$.

If the fluctuation characteristics of the source are described by expressions (3.12) and (3.13), the minimum detectable classical displacement $[x(\tau)]_{min}$ which will cause modulation greater than the fluctuation level will be

$$[x(\tau)]_{\min} = \zeta \frac{(1-R)c}{\pi} \sqrt{\frac{h}{2\nu_0 N_0} A \Delta f}. \tag{3.14}$$

In (3.14) R is the coefficient of reflection from the mirrors, Δf is the frequency band characteristic for $x(\tau)$, ζ is a dimensionless factor, the same as in formula (3.3), c is the speed of light. Formula (3.14) was derived on the assumption that the flux is quite powerful $N_0/h\nu_0\Delta f >> 1$ (under this condition expression (3.13) has sense); it was also assumed that /50 the quantum yield of the photodetector is close to unity.

The $F(\tau)$ force must cause a displacement greater than $[x(\tau)]_{min}$. For example, if $F(\tau) = F_0 \sin \omega_{mech} \tau$ during the interval $\hat{\tau}$, it is necessary that

$$\frac{F_0(\hat{\tau})}{2m\omega_{\text{mech}}} \geqslant [x(\tau)]_{\min} = \zeta \frac{(1-R)c}{\pi} \sqrt{\frac{h}{2N_0\nu_0\hat{\tau}}} A. \tag{3.15}$$

On the other hand, light pressure fluctuations on the optical resonator walls should not be greater than $F(\tau)$. In a case of $F(\tau)$ having the form of a sinusoidal train,

$$F_0 \geqslant \zeta \frac{\sqrt{\overline{\Delta N^2}}}{(1-R)c} = \zeta \frac{1}{(1-R)c} \sqrt{\frac{2h\nu_0 N_0 A}{\hat{\tau}}}. \tag{3.16}$$

From a comparison (3.15) and (3.16) it can be seen that the requirements on source intensity are contradictory: the greater the mean intensity N_0, the smaller are the forces which can be discriminated (see (3.15)); on the other hand, an increase in N_0 leads to an increase in the absolute magnitude of pressure fluctuations on the mirrors, and therefore increases the threshold of the detectable force (see (3.16)). This means that, as in the case of an electronic device, there is an optimum mean intensity $[N_0]_{optim}$ at which the F_0 values computed using (3.15) and (3.16) coincide. This force amplitude will be the minimum detectable quantity.

Solving (3.15) and (3.16) jointly for F_0 and N_0, we obtain

$$[F_0]_{\min} = \zeta \frac{2}{\hat{\tau}} \sqrt{\hbar\omega_{\text{mech}} m A}, \tag{3.17}$$

$$[N_0]_{\text{optim}} = \frac{m\omega_{\text{mech}} (1-R)^2 \lambda_0 c}{\pi \hat{\tau}}. \tag{3.18}$$

In contrast to formulas (3.10) and (3.11), in formula (3.17) the $\hat{\tau}$ parameter is simultaneously the time of the effect and the time expended on measurement. As can be seen from a comparison of (3.10) and (3.17), when $A = 1$ they coincide with an accuracy to a numerical factor of about unity. It is very important that the parameters of the optical resonator do not enter into the formula for the minimum detectable force (other than the factor A, characterizing the source statistics). /51

After comparing formulas (3.17), (3.10) and (3.11), it can be concluded that neglecting the discreteness of the oscillator energy levels is justified in any case when $A \geqslant 1$ and $n \gtrsim 10^2$. These conditions are necessary for the use of the approximating formula (3.17) in computing the minimum detectable amplitudes of forces for a stipulated measurement time. We note the similarity of formulas (3.17) and (3.5): in both cases, for both the optical and electronic sensors, the preliminary information on signal duration $\hat{\tau}$ makes it possible to formulate the law of decrease in the minimum detectable force: $(\hat{\tau})^{-1}$.

In deriving (3.17) and (3.18) we examined an idealized model of an experiment with a test body; accordingly, the region of applicability for these formulas is limited. However, as will be clear from the estimates given below, this region is relatively broad, particularly for small ω_{mech} and large $\hat{\tau}$.

As already mentioned, expression (3.13) is valid only for relatively powerful light fluxes when the condition $N_0/h\nu_0\Delta f >> 1$ is satisfied. Assuming in this inequality that $\Delta f = 1/\hat{\tau}$ and substituting $[N_0]_{optim}$ into it, we obtain the lower limit for masses in experiments with test bodies for which the approximate formula (3.17) is correct

$$m \gg \frac{\pi\hbar}{\omega_{\rm mech}(1-R)^2\lambda_0^2}. \tag{3.19}$$

If it is assumed in (3.19) that $\omega_{mech} = 1\ \mathrm{sec}^{-1}$, $R = 0.99$, $\lambda_0 = 6\cdot10^{-5}$ cm, it is necessary that $m >> 1\cdot10^{-13}$ g. Thus, the lower limit for oscillator masses for which (3.17) and (3.18) are correct is quite small.

In order to be able to use formula (3.16), from which (3.17) and (3.18) were derived, it is necessary that the mechanical oscillator have classical interaction with the light flux pressure fluctuations. This means that the oscillator must have an initial amplitude of mechanical oscillations x_{in}, sufficiently large in order that the light pressure fluctuations during the time $\hat{\tau}$ either draw from it or impart to it an energy substantially greater than $h\omega_{mech}$. Assuming $A \simeq 1$, it is easy to demonstrate that /52

$$x_{\rm in} \gg c(1-R)\sqrt{\frac{\hbar}{N_0\nu_0\hat{\tau}}}. \tag{3.20}$$

In its physical sense the x_{in} parameter is similar to the characteristic amplitude of nuclear oscillations in a crystal lattice (characteristic temperature) in the Mossbauer effect. Expression (3.20) indicates that the

$[x(\tau)]_{min}$ value in (3.14) and (3.15) must be regarded as an increment of the displacement of oscillator mass toward x_{in}.

By substituting the $[N_0]_{optim}$ value from (3.18) into (3.20) in place of N_0, we obtain

$$x_{in} \gg \sqrt{\frac{\pi\hbar}{m\omega_{mech}}}. \tag{3.21}$$

It is easy to see that the condition coincides with the above-mentioned requirement $n \gtrsim 10^2$.

The formulas (3.17) and (3.18) derived above, as well as the restrictions on the region of their applicability (3.19) and (3.21), are valid for the considered experimental model with a test body in which one extremely important simplification has been made: the light flux is the only source of a mechanical fluctuation effect, and the oscillator mass m, together with the mirror of the Fabry-Perot resonator, is regarded as an absolutely solid body. However, the real mass and real mirror have a finite temperature and spectrum of characteristic mechanical frequencies. If the mass and mirror are regarded as ideally heat-insulated from the laboratory, in this case as well the fluctuation exchange of energy between the internal mechanical degrees of freedom in the passive "mass-mirror" thermostat can lead to a swaying of the oscillator as a whole. We will regard this mechanism of the fluctuation effect on an oscillator in a simplified model.

We will assume that the oscillator rigidity K_1 asymmetrically connects the oscillator mass m to the laboratory. We will assume that K_1 does not introduce mechanical friction into the oscillator ($H_1 = 0$), and accordingly, /53
if the mass m is considered an absolutely solid body, the limiting formulas (3.17) and (3.18) are applicable for the oscillator. The characteristic thermal oscillations of the mass m, due to the asymmetrical connection of rigidity K_1 to the laboratory, lead to a swaying of the center of mass relative to the laboratory. It is clear that the principal contribution to such a process must be from low-frequency types of oscillations of the mass m. We will limit ourselves to a consideration of the most low-frequency type, having the frequency ω_2. For this purpose we will visualize the mass m in the form of a

quadrupole oscillator with the concentrated masses $m/2$, rigidity $K_2 = m\omega_2^2/4$, and friction $H_2 = \omega_2 m/2Q_2$ (Figure 12). The fluctuation force F_{fl}, whose spectral density is $(F_{fl})_\omega^2 = 4\kappa TH_2$, causes relative displacements of the masses forming the quadrupole oscillator. Since the left mass of the quadrupole oscillator is connected to the laboratory by the rigidity K_1, whereas the right mass is "free", the center of mass of the quadrupole oscillator will be displaced relative to the laboratory coordinate system. The F_{fl} force has a continuous frequency spectrum beginning from zero; accordingly, the center of mass will be excited and at the frequency $\omega_1 = \sqrt{K_1/m}$.

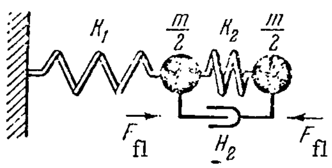

Figure 12

Thus, formulas (3.17) and (3.18) will have validity if the fluctuation increments of the amplitude of oscillations for the center of mass m, caused by pressure fluctuations $[N_0]_{optim}$, exceed the displacements caused by the F_{fl} fluctuation force related to H_2. In the case of a sinusoidal train it is necessary that

$$\frac{2}{\tau}\sqrt{\hbar\omega_1 m} \gg \frac{\omega_1^2}{\omega_2^2}\sqrt{\frac{2\varkappa T m\omega_2}{Q_2\tau}} = \frac{\omega_1^2}{\omega_2^2}\sqrt{\frac{4\varkappa T H_2}{\tau}}. \qquad (3.22)$$

In deriving condition (3.22) it was taken into account that $\omega_1 \ll \omega_2$.

A more rigorous examination of this problem, taking into account all possible types of oscillations in the mass m which can lead to a displacement of the center of mass m, leads to the following condition: /54

$$\frac{2}{\tau}\sqrt{\hbar\omega_1 m} \gg \sum_{i=2}^{\infty}\frac{\omega_1^2}{\omega_i^2}\sqrt{\frac{4\varkappa T\theta L}{\tau}}, \qquad (3.23)$$

where θ is the distribution of friction along m, L is the linear dimension of the mass in the direction of oscillations. As can be seen from a comparison of (3.22) and (3.23), it is entirely possible to limit oneself to the first term of the series, due to its rapid convergence.

It follows from the above discussion that formulas (3.17) and (3.18) can be used and there is no need to employ an absolutely solid body as a test body. It is sufficient that in the case of finite $\hat{\tau}$ there be a small ratio of the fundamental oscillator frequency to the frequency of the lowest type of oscillations of the test body (or the test body together with a mirror).

Now we will give estimates for specific parameters entering into condition (3.22). If m = 10 g, $\omega_1 = 6 \cdot 10^{-1}$ sec^{-1}, $\hat{\tau} = 10^3$ sec, for the left-hand side of (6.22) we obtain $1.6 \cdot 10^{-16}$ dyne. If the mass m_1 = 10 g has the form of a sphere fabricated from ordinary materials (steel, quartz, aluminum), then $\omega_2 \simeq 2 \cdot 10^6$ sec^{-1} and $Q_2 \simeq 10^4$. Assuming T = 300 °K, for the right-hand side of (3.22) we obtain $3.6 \cdot 10^{-20}$ dyne. Thus, for the above-mentioned parameters condition (3.22) is satisfied. In other words, even at room temperature the fluctuations of quantum origin in an optical indicator determine the minimum detectable force.

In conclusion of our analysis of formulas (3.17) and (3.18) we note that the use of optimum strategy (which makes it possible to decrease $[F_0]_{min}$ as $(\hat{\tau})^{-1}$) is limited: on the right-hand side of condition (3.22) $\hat{\tau}$ appears as $(\hat{\tau})^{-1/2}$, whereas on the left-hand side, it appears as $\hat{\tau}^{-1}$. This means that with sufficiently large $\hat{\tau}$ and with the other parameters fixed, condition (3.22) will be impaired and the use of an optimum measurement strategy will be impossible.

Now we will briefly discuss still another restriction for formulas (3.17) and (3.18) in the direction of high frequencies of characteristic oscillations for a mechanical oscillator.

A light impulse with the energy ΔW (the impulse can also be of fluctuation origin) upon incidence on the oscillator, causes not only the mechanical /55
impulse ΔW $(1 + R)c^{-1}$, but also partial heating of the mass m, and therefore, a nonstationary temperature field. This field leads to the appearance of acoustic waves in the mass m; these waves can also sway the oscillator as a whole. The appearance of thermoelastic waves under the influence of powerful light impulses has already been observed experimentally [26-28]. It can be shown that the ratio of amplitudes of oscillator oscillations caused by the

light pressure a_{light} and the thermoelastic effect a_{te} conforms approximately to the following expression:

$$\frac{a_{\text{light}}}{a_{\text{te}}} \simeq \frac{(1+R)\chi}{(1-R)L\alpha c\omega_{\text{mech}}}. \tag{3.24}$$

Here R is the reflection coefficient, χ is the heat capacity of material in the mass, α is the coefficient of thermal expansion, L is of the order of the linear dimensions of the mass m (for further details, see [29]).

It follows from expression (3.24) that in the case of sufficiently high ω_{mech} (of about several kc/sec for steel, quartz, and aluminum) the oscillations of a mechanical oscillator caused by fluctuations of light pressure in the flux incident on the oscillator will be comparable to the oscillations caused by the thermoelastic effect. Thus, expression (3.24) limits the applicability of (3.17) and (3.18) for mechanical oscillators with a relatively high ω_{mech} value.

The computations made above for a Fabry-Perot resonator, used as a small displacement indicator, lead to formula (3.17) for the minimum detectable force, in which only the factor A, dependent on the statistical characteristics of the optical source, was included. Other parameters for the optical indicator were not included in the expression for $[F_0]_{min}$. This important circumstance is not correct for the Fabry-Perot resonator alone. Without derivation, we will give the results of similar computations for another variant of an optical indicator. We will assume that instead of a linear mechanical oscillator we employ a torsional oscillator with the moment of inertia I and the characteristic frequency ω_{mech} (Figure 13). As the indicator we used the so-called "knife and slit" (optical lever) method. Small /56 oscillations of a mirror attached to the mechanical oscillator cause a displacement of the focal spot of the optical ray passing through the lens O. The presence of a fixed, attached optical knife near the focal plane results in an intense light flux modulation for small angular oscillations of the oscillator; this is registered by a photodiode.

If it is assumed, as in the derivation of (3.17), that the only source of a fluctuation effect on such an oscillator is light pressure fluctuations on the mirror, by making computations similar to those given for (3.17), it is possible to derive an expression for the minimum detectable moment $[\text{Mom } F(\tau)]_{min}$ for the optimum intensity $[N_0]_{optim}$ of the light flux. If the moment of force has the form of a sinusoidal train, then

$$[\text{Mom } F_0]_{\min} \simeq \zeta \frac{2}{\tau} \sqrt{\hbar\omega_{\text{mech}} I A'}, \tag{3.25}$$

$$[N_0]_{\text{optim}} = \frac{I\omega_{\text{mech}} \lambda_0 c}{\tau a^2}, \tag{3.26}$$

where $a = Db/2f$, D is the lens aperture, f is the lens focal length, b is the distance from the mirror to the focal spot. The coefficient A' coincides with the coefficient in formula (3.13).

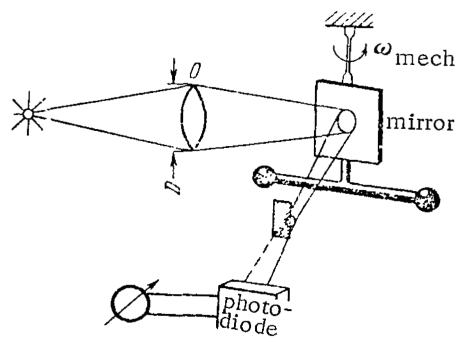

Figure 13

It is important that the requirements on source frequency stability in such a model are substantially lower than in the experiment with a Fabry-Perot resonator. /57

Thus, formulas (3.25) and (3.17) coincide with the replacement of the mass by the moment of inertia and the force by the moment of force. Formulas (3.25) and (3.26) for the minimum detectable moment and optimum intensity have the same limitations as (3.17) and (3.18).

§ 4. Optimum Strategy for Measurements in Experiments With Test Bodies; Potential Resolution.

Now we will compare different fluctuation limitations on the detection of a small force acting upon a macroscopic mechanical oscillator.

As was demonstrated in §§ 1 and 3, it is possible to obtain three different threshold formulas for the minimum detectable forces. We will compare the formulas for the amplitudes of the minimum detectable forces (the force has the form of a resonant sinusoidal train) for these three threshold cases

$$[F_0]_{min} = \zeta \sqrt{\frac{4\varkappa T m}{\hat{\tau}\tau^*}}, \tag{1.15}$$

$$[F_0]_{min} = \zeta \frac{4}{\hat{\tau}} \sqrt{\pi \sqrt{2} \frac{\varkappa T m \omega_{mech}}{\Omega_{cir}}}, \tag{3.5}$$

$$[F_0]_{min} = \zeta \frac{2}{\hat{\tau}} \sqrt{\hbar m \omega_{mech} A}. \tag{3.17}$$

Formula (1.15) determines the lower limit for the detectable amplitude F_0 in the case of classical thermal noise, provided the fluctuation effect from the measuring instrument is substantially less than the effects of a dissipative element connecting the test mass to the laboratory. Formulas (3.5) and (3.17) make it possible to estimate the minimum detectable amplitude of the classical force F_0, provided there is optimum tuning of the electronic (formula (3.5)) or optical (formula (3.17)) sensors, which are the only sources of the fluctuation effect on m. The extremal formulas (3.5) and (3.17) are correct only for not excessively high ω_{mech} and sufficiently large m (for further details, see § 3). The optimum tuning method, which leads to formulas (3.5) and (3.17), as was emphasized in the preceding section, is not the only possible method. The absence of the detector parameters in explicit form in the formulas for $[F_0]_{min}$ is evidently universal for any types of detectors with optimum tuning. /58

Comparison of (3.5) and (3.17) shows that optical indicators of small displacements in theory make it possible to obtain (with optimum tuning) lesser threshold values for the detectable force.

We will note still another important circumstance for cases when formulas (3.5) and (3.17) are applicable. In contrast to (1.15), in formulas (3.5) and (3.17) the dispersion for a single measurement decreases as $(\hat{\tau})^{-2}$. Since the parameters A and T are not always known in advance, it is necessary to repeat the measurements so that instead of the dispersion it will be possible to use its evaluation from a small sample (a similar examination for formula (1.15) was given in § 1 for the unknown τ^* and T). If we are not concerned with statistical errors of the second kind, then $\zeta = t_{1-\alpha}(n)$ and the $[F_0]_{min}$ value is proportional to

$$t_{1-\alpha}(n)\,[\hat{\tau}\sqrt{n}]^{-1} = \sqrt{n}\,t_{1-\alpha}(n)\,\tau_{\mathrm{meas}}^{-1}$$

where the total time expended on the measurement $\tau_{meas} = n\hat{\tau}$, n is the number of repeated measurements, $t_{1-\alpha}(n)$ is the quantile of Student's distribution. The product $\sqrt{n}\,t_{1-\alpha}(n)$ has a minimum dependent on the selected value of the statistical error of the first kind α. It is easy to determine this minimum by using well-known tables of $t_{1-\alpha}(n)$. For example, for $\alpha = 0.01$, $\min[\sqrt{n}\,t_{1-\alpha}(n)] = 9.8$ when $n = 6$. Thus, by stipulating the α value it is possible to select the optimum n for the available measurement time τ_{meas}. This n corresponds to the minimum $[F_0]_{min}$ when the statistical characteristics of the source (T or A) are unknown. It is clear that an optimum strategy is possible in selecting n and in taking into account the statistical errors of the second kind.

These expressions for $[F_0]_{min}$ for a specific case when the force has the form of a resonance sinusoidal train with the duration $\hat{\tau}$, as is easy to see, also retain validity in a case when $F(\tau)$ has the form of a short impulse $\hat{\tau} \ll 1/\omega_{mech}$. In this case there is a slight change only in the numerical factors on the right-hand sides of (1.15), (3.5) and (3.17). It is easy to repeat the computations leading to formulas (3.5) and (3.17), for a $F(\tau)$ force of an arbitrary form which is finite in time. /59

It is possible to give preference to formula (1.15) or (3.5) and (3.17) when discussing specific experimental conditions only if data are available for T, τ^*, Ω_{cir}, ω_{mech} and A. Obviously, in the case of sufficiently large τ^* the response limit will be determined by the instrument fluctuation effect (formulas (3.5) or (3.17)).

As already mentioned in § 3, formulas (3.5) and (3.17) coincide with an accuracy to the numerical factor $2\sqrt{\pi\sqrt{2}A}$, provided that $\kappa T/\Omega_{cir}$ is replaced by the Planck constant $\hbar$. However, (3.5) is correct only when $\kappa T/\Omega_{cir} \gg \hbar$. In further numerical estimates we will assume in (3.5) that $\kappa T/\Omega_{cir} = 10\hbar$ (this will correspond to $A \simeq 10^2$ in formula (3.17)).

Now we will cite examples of the attainable response in different types of instruments. In the preceding section, in the numerical example for (3.5) when $\kappa T/\Omega_{cir} = 10\hbar$, we assumed $m = 1$ g, ω_{mech} 1 sec^{-1}, $\hat{\tau} = 1 \cdot 10^3$ sec, $\zeta = 2$ (this corresponds to a reliability of about 0.95) and we obtained $[F_0]_{min} = 1.6 \cdot 10^{-15}$ dyne. In particular, this means that the minimum periodic acceleration which can be registered in the case of a mass $m = 1$ g during the time $\hat{\tau} = 10^3$ sec and with a period of change in acceleration $\tau_0 \simeq 6$ sec is $1.6 \cdot 10^{-15}$ cm/sec^2; this is substantially less than the quantity which can be resolved at the present time.

If it is assumed that the F_0 force was caused by presence of an electric charge q in a body with the mass m and the effect of an electrostatic field with the strength $E(\tau) = E_0 \sin \omega_{mech} \tau$, the minimum electric charge $[q]_{min}$ which can be detected in a body with a mass of 1 g for the above-mentioned $\hat{\tau}$, ω_{mech} is $1.6 \cdot 10^{-17}$ CGSE, assuming that $E_0 = 10^2$ CGSE (i.e., 30 kV/cm). We recall that the electron charge is $4.8 \cdot 10^{-10}$ CGSE.

After making a similar estimate of the detectable magnetic field strength /60 and the electric current intensity (it is convenient to use formula (3.25) applicable to a magnetometer and galvanometer for this purpose), it is easy to see that for these parameters as well, like for forces, charges, and accelerations, there is an enormous unexploited resolution: 7-10 orders of magnitude in comparison with that already attained. It is extremely important to take into account the theoretically detectable quantities for forces, accelerations, etc. in any discussion of the possibilities of carrying out precise physical experiments. This discussion will be presented in greater detail in Chapters 2 and 3.

In supplementing the above numerical example, we will evaluate the conditions under which formulas (3.5) and (3.17) can be used. Substituting $[F_0]_{min} = 1.6 \cdot 10^{-15}$ dyne into the left-hand side of (1.15), and $\hat{\tau} = 10^3$ sec, $m = 1$ g into the right-hand side of (1.15), we obtain $\tau^* \simeq 2.5 \cdot 10^{16}$ sec. Thus, the estimates given above are correct if the relaxation time τ^* exceeds $2.5 \cdot 10^{16}$ sec. For a small sphere with the radius 1 cm and the mass 1 g at $T = 300$ °K this is possible in an oxygen atmosphere only with a concentration of molecules less than $\tilde{f} \simeq 20$ cm^{-3}. We recall that the τ^* values obtained until now in terrestrial laboratories for more massive bodies did not exceed 10^9 sec. Thus, for attaining the response corresponding to formulas (3.5) and (3.17) it is necessary either to be able to increase τ^* sharply under terrestrial conditions, or else carry out experiments on orbiting stations.

Now we will discuss the longer-range possibilities of increasing response in experiments with test bodies. As is clear from the derivation of formula (3.17), it is essentially a classical formula for a mechanical oscillator (although it follows from the existence of fluctuations of quantum origin in an optical source). The relation of uncertainties corresponds to formula (3.11), from which it follows that in order to increase response it is desireable to increase the initial amplitude of the oscillations. After comparing (3.11) and (3.17), it can be concluded that the only method for attaining the threshold determined by the relation of uncertainty for a mechanical oscillator is a decrease in the A value. Such a possibility theoretically exists in lasers with a sufficiently rigorous limiting cycle. If we analyze well-known expressions for the spectral density of fluctuations of amplitude M_f^2 (for example, see [25]), it can be seen that with a sufficiently great rigor of the limiting cycle it is possible to attain M_f^2 values less than $2h\nu/N_0$. However, until now present-day lasers have $A > 1$. The development of a nonlinear optical system (by analogy with the classical fluctuation dampers) will evidently make it possible to decrease the A value. /61

Now we will briefly mention still another area of possible applicability of experiments with test bodies in physical investigations. We will return to the numerical example considered above. The force impulse $\hat{F}\hat{\tau} = 1 \cdot 10^{-13}$ dyne·sec can be registered in accordance with formula (3.17), provided that it is assumed that $m = 1$ g, $\omega_{mech} = 1$ sec^{-1}, $A = 10$, $\zeta = 2$. Such an impulse is characteristic of an electron having an energy of 100 MeV. This means that at least in theory mechanical oscillators with small friction can serve as detectors for registering high-energy elementary particles. It is interesting to note that such a detector is not a "virtually unstable system" (D. I. Blokhintsev [30]) like many of the well-known detectors.

In summarizing the examples and computations presented in this chapter, it should be emphasized once again that there is an extremely great unexploited reserve of response in experiments with test bodies. The possibility of employing this response reserve in different types of experimental investigations will be discussed in greater detail in the sections which follow.

CHAPTER II /62

FUNDAMENTAL EXPERIMENTS WITH TEST BODIES

In selecting the material included in this chapter, the author gave preference to experiments which have been made during recent years and which play a substantial role for a number of fundamental physical concepts. On the other hand, these experiments make it possible to judge the present-day level of technology in this field of physical measurements. In performing these experiments some interesting methods were used which can be useful to experimenters.

In each of the three sections in this chapter, we analyze the possibilities of increasing resolution in accordance with the theoretical concepts concerning threshold response set forth in Chapter I.

§ 5. Checking the Equivalence Principle

The Equivalence Principle. A. Einstein noted (see review [31]) that in his opinion further experimental refinement of the equivalence principle (weak equivalence principle) is more important than new checkings of the agreement between the computed and observed secular displacements of the perihelion of Mercury and deflections of a light ray in the sun's gravity field. The weak equivalence principle (constancy of the ratio of an inert mass to a gravitational mass for different bodies) has been checked repeatedly. The first check of this fundamental principle, later serving as the basis for the general theory of relativity, was made by Newton. Later the correctness /63
of the weak equivalence principle was refined several times (Bessel, Eotvos, Zeeman, Southerns). The most precise checking of this theory was made during 1959-1963 by Dicke, in collaboration with a group of colleagues. They established that the relative difference in accelerations of free falling in the sun's gravity field for two masses fabricated from aluminum and gold does not exceed $\sim 1 \cdot 10^{-11}$. This means that the relative difference in the m_{in}/m_{grav} ratio for aluminum and gold is also not greater than $1 \cdot 10^{-11}$ [32].

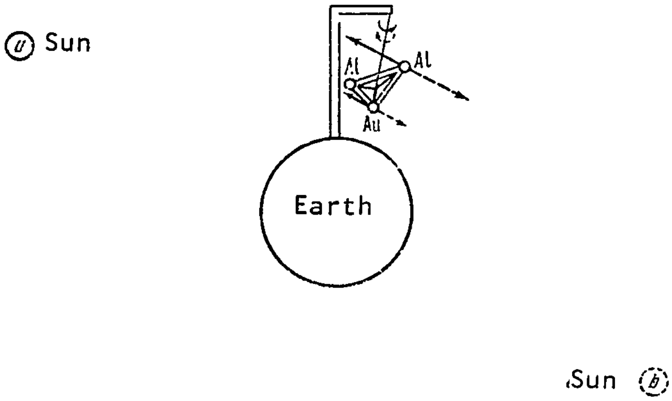

Figure 14

Now we will discuss in greater detail the method used in this experiment; as will be clear from the text which follows, it is undoubtedly one of the most precise and exhaustive of the investigations made during recent years. Figure 14 is a block diagram of the apparatus. A triangular platform of fused quartz was suspended on a fine quartz filament. Weights of approximately equal mass were attached to the ends; two of these were fabricated from aluminum and one from gold. The earth, together with the apparatus, is in a state of free fall in the sun's gravity field (~ 0.6 cm/sec^2). As a result of the earth's diurnal rotation, the possible difference in the accelerations of aluminum and gold (if the weak equivalence principle is not /64
precisely satisfied) creates a periodic moment of force applied to the platform. The period of change in this moment will be equal to one day. If the sun is situated in the position *a* relative to the earth and if the acceleration of aluminum is greater than for gold, the moment of force will be directed counterclockwise. However, if the sun is in the position *b*, it will be clockwise. Thus, this group of experimenters (in contrast to Eotvos[1]) had to detect a small moment of force, changing with time in conformity to a sinusoidal law, applied to the platform.

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[1] The method employed by Eotvos in his experiment carried out about 50 years ago is described in detail in [22].

Gold and aluminum are widely separated in the periodic table of elements. The ratio of the number of neutrons to the number of protons is 1.08 for aluminum and 1.5 for gold. Virtually all the electrons in the aluminum atom are nonrelativistic, whereas for gold the mass of electrons close to the nucleus is approximately 15% greater than for nonrelativistic electrons. The relative mass defect for the gold nucleus differs substantially from the mass defect for an aluminum nucleus.

In order to attain a resolution in accelerations of mass of about $6 \cdot 10^{-12}$ cm/sec^2, it was necessary to create a sensitive system for detecting small angles of rotation of the quartz platform. Its block diagram is shown in Figure 15. One of the quartz platform surfaces served as a mirror. A weak beam of light from the narrow slit of an optical collimator was reflected from the mirror and illuminated a small oscillating wire. The lens in front of the mirror made it possible to match the actual image of the filament with the wire, whose oscillations occurred in the plane perpendicular to the light flux direction. The light flux modulated in this way was incident on the photomultiplier. The variable components of the photomultiplier current were amplified. The wire was swayed by an electric voltage from a bridge generator at a frequency of 3 kc/sec. If the slit image was symmetric relative to the oscillating wire, only the harmonic of the frequency 3 kc/sec could be observed at the photomultiplier output. Small platform rotations resulted in a displacement of the slit image relative to /66
the wire and the appearance at the photomultiplier output of a variable current with a frequency equal to the frequency of wire oscillations and with an amplitude proportional to the image displacement, and therefore proportional to the platform angle of rotation. The phase of this electric voltage was dependent on the rotation direction. After passing through a narrow-band amplifier, also tuned to a frequency 3 kc/sec, the electric signal from the photomultiplier was fed to one of the phase detector inputs. An electric voltage from the bridge generator was fed to its other input, after amplification and phase correction. Thus, the constant voltage at the phase detector output was proportional to the platform angle of rotation and the sign was determined by the rotation direction. Then, after filtering, the

electric voltage was fed to an automatic tape recorder. This apparatus could measure an amplitude of platform angular displacements of $\sim 1 \cdot 10^{-9}$ rad at a frequency corresponding to the period of the earth's rotation. We note that the resolution attained at such a low frequency approximately corresponds to the record resolution obtained by Jones using a special optical lever (for further details, see Appendix). For a period of characteristic pendulum platform oscillations of about 400 sec this angular resolution corresponds to an amplitude of acceleration of the ends of the platform approximately equal to $6 \cdot 10^{-12}$ cm/sec^2, or its equivalent, measurement of the relative difference in accelerations of the platform ends of $1 \cdot 10^{-11}$ relative to acceleration in the sun's gravity field (0.6 cm/sec^2).

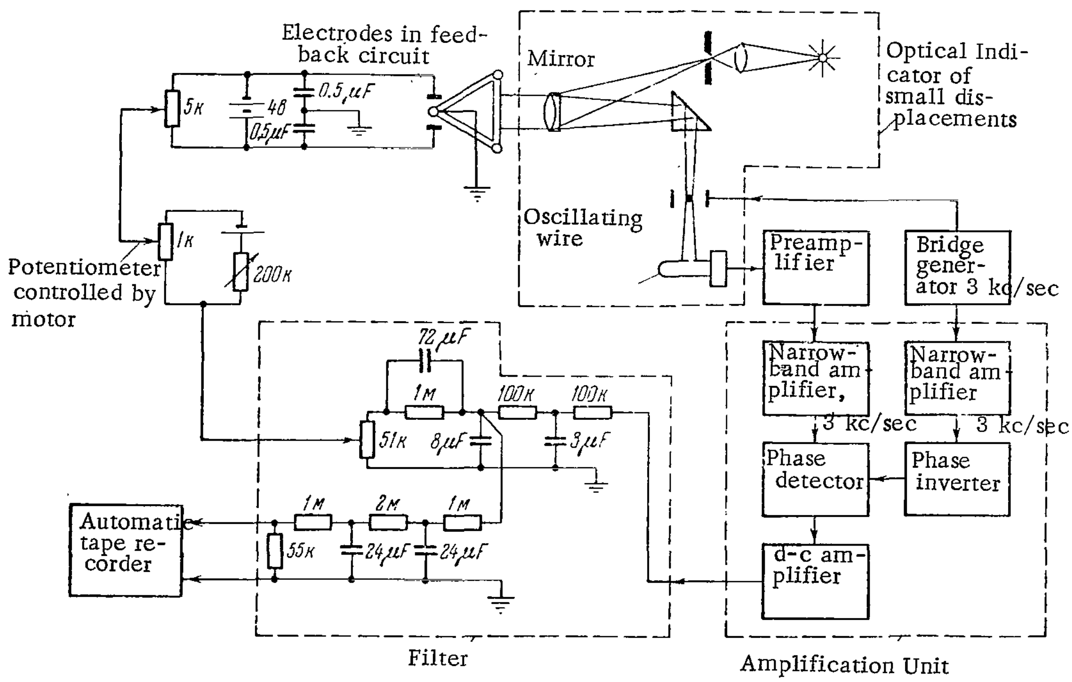

Figure 15

The electric signal from the phase detector output was not fed to an automatic recorder, but to a feedback circuit, which introduced damping into the torsional oscillations of the platform and slightly decreased the period of characteristic oscillations (see Figure 15). The force introducing attenuation into platform motion was created by the Coulomb attraction of one of the masses attached on the platform to two electrodes connected through a filter to the phase detector output. The magnitude of the introduced damping could be regulated by a remote potentiometer, controlled by a motor. The use of this method was dictated by the following important circumstance. If the /67
system for detecting small platform rotations reproduces them strictly linearly on an automatic recorder, there is no need to introduce damping. However, if normal mechanical modes of oscillations of the torsional pendulum are excited in some way, the presence of nonlinearity in the registry system leads to an apparent appearance of a low-frequency signal whose period will be approximately equal to the damping time of these modes. However, the damping time for the fundamental torsional mode, in Dicke's opinion, could attain two years. Accordingly, any seismic interference which is localized in time, with the presence of nonlinearity in the detection system, could be detected as the appearance of a moment of force caused by the difference in accelerations of the aluminum and gold masses.

We will enumerate a number of other methods used by the author of this experiment:

1. In order to attenuate the possible effects of gravitational perturbation introduced by the observer's mass, three test masses were used; this considerably decreased the quadrupole moment of masses. In the experiments by Eotvos only two masses were employed; Dicke feels that the observer must introduce a gravitational perturbation at the level of the attained sensitivity.

2. For this same purpose all the measurements were made by remote control.

3. The elimination of the convection effect near the test masses and the platform was achieved by placing the entire apparatus in an evacuated housing (vacuum $\sim 10^{-8}$ mm Hg).

4. In order to decrease temperature fluctuations, the entire apparatus was placed in a shaft 3.6 m deep; this was covered with a thermal "plug" 1.2 m high. The measurements were made without opening the "plug" for months. At the same time, remote measurements were made and a continuous record of the temperature change at different points on the housing was kept.

5. The pendulum was fabricated from nonmagnetic materials, in such a way as to eliminate the force effect of daily variations in the earth's magnetic field.

6. Fused quartz, covered with a thin layer of aluminum, was used as the suspension filament; this made it possible to ground the platform and weights. In addition, the steplike drift of the quartz filament was considerably less /68 than for tungsten filaments used in the first Dicke series of experiments [32].

7. The rotation of the torsional pendulum was recorded automatically and continuously, and discrimination of a signal with a diurnal period was accomplished using an electronic computer.

Now we will summarize the results. Evidently, the most important result of the experiment must be considered the following: the authors were able to extend the limit to which the weak equibalence principle is satisfied from $5 \cdot 10^{-9}$ (the resolution attained by Eotvos) to $1 \cdot 10^{-11}$; in other words, the accuracy was increased by a factor of 500. However, evidently for the most part due to the presence of a significant nonlinearity in the registry system, it was not possible to attain a resolution corresponding to Brownian fluctuations.

If one takes into account the period of pendulum oscillations (400 sec), the time spent on the measurement (several months), as well as the masses of the test bodies (a few grams), it is easy to compute the resolution level which can be attained in this sort of experiment under terrestrial conditions, assuming that the only source of fluctuations is thermal. Assuming the relaxation time to be equal approximately to the period of oscillations

τ_0, it is easy to find (see § 1) that when $\tau_{meas} \simeq 10^7$ sec it would be possible to detect a difference in the accelerations of test masses of about $1 \cdot 10^{-13}$ cm/sec^2, i.e., while retaining the Dicke method, make more precise (or refute) the weak equivalence principle at the level about $1 \cdot 10^{-13}$. However, if large τ^* values are used (see expression (1.15)), which can be obtained using thin quartz filaments in a vacuum, in terrestrial laboratories it would be possible to advance the limit by still another order of magnitude.

We note still another important circumstance. The seismic interference which substantially hindered the experiment described above is not noise in the usual sense, since it is completely determined (it can be measured simultaneously and independently); accordingly, in the case of a detector of sufficiently high quality, it can be completely excluded from consideration.

Now we will estimate the theoretically attainable level of resolution when checking the weak equivalence principle in experiments with test bodies. In this evaluation we will use formula (3.17). Dividing the right- and left- /69
hand sides by the mass m of the test body, we obtain an expression for the minimum amplitude of the detectable acceleration $[a_0]_{min}$:

$$[a_0]_{\min} = \zeta \frac{2}{\hat{\tau}} \sqrt{\frac{\hbar \omega_{\text{mech}} A}{m}} \tag{5.1}$$

We recall that ω_{mech} is the characteristic frequency of a mechanical oscillator, A is a dimensionless factor characterizing statistical fluctuations at the source, ζ is a factor of the order of several units, dependent on the level of detection reliability, $\hat{\tau}$ is the duration of the effect, having the form of a sinusoidal train with a frequency ω_{mech}. Formula (5.1), like (3.17), is correct only in the case of optimum tuning of the optical detector, being the only source of a fluctuation effect on the test mass. Assuming in (5.1) that $\omega_{mech} = 2\pi \cdot 10^{-5}$ sec^{-1} (about the frequency corresponding to the earth's period), $m = 10^3$ g, $\hat{\tau} = 10^7$ sec (i.e., approximately the same as in the Dicke experiments), $A = 10$, $\zeta = 2$, we obtain $[a_0]_{min} = 1 \cdot 10^{-23}$ cm/sec^2. If the experiment is carried out in accordance with the Dicke method and the accelerations of two masses are compared in the solar gravity field

($a \simeq 0.6$ cm/sec^2), it would be possible to check the equivalence principle to a relative accuracy about $1 \cdot 10^{-23}$. It is interesting to note that in this way it is possible to attain a resolution at least at the level of the dimensionless weak correlation constant:

$$\left(\frac{\gamma m_\pi^2 c}{\hbar^3}\right)^2 = 3 \cdot 10^{-14}, \tag{5.2}$$

where m_π is the pion mass, γ is the gravitational constant.

It is important to note that at the present time there are no theoretical premises which would indicate the existence of any threshold level below which the weak equivalence principle could cease to be satisfied.

In concluding this section, we will discuss a variant of an experiment for checking the weak equivalence principle in space on a low-flying earth satellite. We will assume that we have an earth satellite in a nearly /70
circular orbit (Figure 16). Assume that the satellite has the shape of a thin toroid with the radius r, whose two halves m_1 and m_2 are fabricated from different substances (similar to the choice in the Dicke experiment). We will also assume that $m_1 \simeq m_2$. If the plane of the toroid coincides with the orbital plane and the period of rotation is equal to the period of satellite revolution around the earth, provided that the weak equivalence principle is precisely satisfied, the extension of the line ab at the satellite should always be directed toward the center of the earth. However, if it is only approximately satisfied and there is a small difference Δa in the accelerations m_1 and m_2, during the time $\hat{\tau}$ the satellite rotates by an angle $\Delta\phi$, equal to:

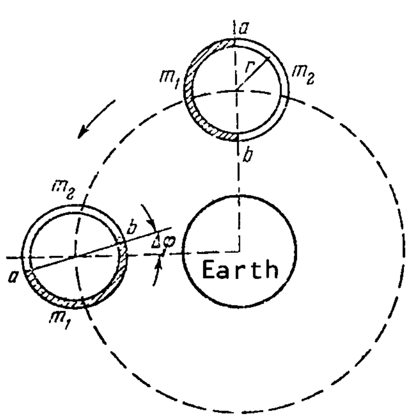

Figure 16

$$\Delta\varphi \simeq \frac{\Delta a \tau^2}{2\pi r}. \qquad (5.3)$$

Assuming that $\Delta a = 3 \cdot 10^{-12}$ cm/sec^2, $\hat{\tau} = 2 \cdot 10^5$ sec (about 2.3 days), $r = 5$ cm, we obtain $\Delta\phi = 4 \cdot 10^{-3}$ rad = 0.24°, an easily measureable value. Expression (5.3) is correct only for small $\Delta\phi$. If the satellite orbit is situated at a relatively low altitude from the earth's surface (about 1,000 km), the gravitational acceleration is $g_{♁} = 7 \cdot 10^2$ cm/sec^2, and in accordance with this estimate, it would be possible to check the equivalence principle with an accuracy to $\Delta a / g_{♁} \simeq 5 \cdot 10^{-15}$.

Now we will briefly list the principal requirements whose satisfaction is necessary for attaining such an accuracy. In order for expression (5.3) to be satisfied (when $\Delta a \neq 0$) and $\Delta\phi$ to increase quadratically with time, it is necessary to satisfy the inequality $\hat{\tau} \ll \tau_0$, where τ_0 is the period of characteristic oscillations of the satellite in the earth's nonuniform gravity field. The period of oscillations will be finite, provided that the quadrupole moment of masses of the toroid is nonzero. Assuming a nonuniformity in the distribution of mass in the toroid, equal to $\Delta m/m$, it is easy to obtain /71

$$\tau_0 \simeq 2\pi \sqrt{\frac{m}{\Delta m} \cdot \frac{R^3}{2\gamma M_{♁}}}, \qquad (5.4)$$

where R is the distance from the satellite to the center of the earth, γ is the gravitational constant, $M_{♁}$ is the earth's mass. If $\Delta m/m = 1 \cdot 10^{-5}$, $R = 7 \cdot 10^8$ cm, then $\tau_0 = 1.3 \cdot 10^6$ sec, which does not contradict the condition $\hat{\tau} \ll \tau_0$ for the above-mentioned estimate.

Geometrical inaccuracy in fabricating the toroid can have the following result: the light pressure of solar radiation will impart to the toroid an acceleration which can simulate impairment of the weak equivalence principle. If it is assumed that the total mass of the toroid is about 10^4 g and the reflection coefficient for its surface is about 0.9, then with an inaccuracy in fabricating its surface of $\Delta s/s$ about 10^{-5}, the linear acceleration of the toroid caused by solar pressure will be about $1 \cdot 10^{-13}$ cm/sec^2, i.e., less than the estimate given above by an order of magnitude.

Strong equivalence principle. The independence of the laws of physics on the presence or absence of a uniform gravity field is usually called the strong equivalence principle. This principle has not been subjected to serious experimental checking. Dabbs, in collaboration with a group of colleagues [33], carried out an experiment for measuring the acceleration of free falling $g_{\text{♁}}$ of a beam of neutrons initially directed horizontally in the earth's gravity field. It was established in this experiment that the difference in Δg values for different neutron spin orientations, if it exists, does not exceed the relative value $\Delta g/g_{\text{♁}} \simeq 1 \cdot 10^{-2}$.

Morgan and Peres [34] demonstrated that the absence of an influence of nuclear spin orientation on the level of mass defect in experiments of the Eotvos-Dicke type should serve as confirmation of the strong equivalence principle. Since dynamic methods have now been successfully developed for orientation of nuclei, making it possible to have 60-70% of the nuclei oriented in a selected direction [35], the accuracy in checking the strong /72
equivalence principle can evidently be the same as for the weak equivalence principle, provided the proposal of Morgan and Peres is adopted.

L. I. Slabkiy, V. K. Martynov, and the author [36] undertook an experiment to determine the upper limit of the possible influence of nuclear spin orientation on the weight of a test body. We established the absence of such an influence, at least at the level $6 \cdot 10^{-10}$ (for further details see [36]). In the experiments we employed the method of discriminating a signal from noise as described in § 1. The relatively low resolution was entirely determined by the degree of uniformity of the magnetic field used in orienting the nuclei.

§ 6. Quantum Macroscopic Effects

Relatively recently so-called quantum macroscopic effects were predicted and discovered: formation of vortical filaments in superfluid helium (Winen [37]) and quantizing of the magnetic flux in hollow superconducting cylinders (Deaver, Fairbank [38], Doll and Näbauer [39], also see the review [40]).

Now we will discuss the method employed by Doll and Näbauer for investigating the possible strength of the magnetic flux in a small superconducting cylinder. The essence of this phenomenon is as follows. The magnetic flux created by a field current through the inner cavity of the cylinder is quantized:

$$\Phi = \pi R^2 B = \Phi_0 n = \frac{\pi \hbar c}{e} n, \tag{6.1}$$

where Φ is the magnetic flux, B is the field within a cylinder whose internal radius is R, e is the electron charge, c is the speed of light. The parameter n assumes only a whole-number value: $n = 0; \pm 1; \pm 2, \ldots$ The numerical value of the "magnetic flux quantum" is $\Phi_0 = 2.06 \cdot 10^{-7}$ gauss·cm^2. In order for the field value B to be comparable to the earth's field and in order thereby to avoid local magnetic field fluctuations near the apparatus, Doll and Näbauer used a cylinder with a small internal cross section.

Figure 17 is a diagram of the experiment. A lead cylinder with the length $l = 6 \cdot 10^{-2}$ cm was sprayed on a quartz rod 10 μm in diameter. The rod was suspended horizontally on a torsional suspension in such a way that its axis was directed perpendicular to the magnetic field created by the coil D_1. A sensitive optical indicator made it possible to register the small torsional pendulum oscillations. /73

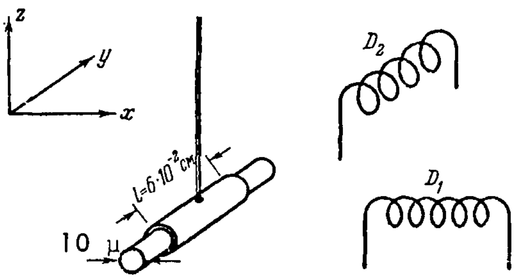

Figure 17

If within the cylinder the flux is Φ_y, and the coil field D_1 is equal to B_x, the moment of force applied to the cylinder is

$$\text{Mom}\, F = \frac{\Phi_y B_x l}{4\pi}. \tag{6.2}$$

In the experiment the magnetic field B_x was 10 oe. With Φ_y = $2.06 \cdot 10^{-7}$ gauss·cm^2 and $l = 6 \cdot 10^{-2}$ cm it was therefore necessary to measure a moment of force of about $1 \cdot 10^{-8}$ dyne·cm. The sequence of measurements was as follows:

a) The lead cylinder was heated to a temperature above the critical temperature for lead, after which the coil D_2 was cut in, creating a permanent magnetic field B_x along the cylinder axis.

b) The temperature was reduced below the critical value and then the coil D_2 was cut out. Thus, some magnetic flux Φ was "frozen" in the lead cylinder.

c) The coil D_1 was cut in for measuring the strength of this flux; the /74 coil created a magnetic field B_x = 10 oe in the neighborhood of the pendulum. This caused the appearance of a torsional moment (6.2), applied to the pendulum. The field in the coil D_1 was reversed automatically in rhythm with the oscillations. Thus, the frequency of the variable moment was precisely equal to the resonance frequency[1]. The strength of the magnetic flux frozen in the lead cylinder was determined from the transient amplitude of the oscillations. Then the pendulum was heated to a temperature above the critical value and the procedure described above was repeated, but with a different field B_y.

It was found from these measurements that when the field B_y is less than 0.1 oe the magnetic flux within the cylinder is equal to zero; when the B_y /75 field is from 0.1 oe to about 0.2 oe the magnetic flux remains constant, and then increases in a jump (Figure 18). The magnitude of the field flux "step" agreed well with (6.1) (relative error of about 20%). It can be seen from the above that the experiment carried out by Doll and Näbauer should be included in the group of experiments with test bodies.

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[1] It is interesting to note that this same method for automatic frequency trim was used in the classical study by Einstein and de Haas [41].

Now we will estimate the strength of the minimum frozen-in flux $[\Phi]_{min}$ or the flux increment which can be detected in such an experiment when using optimum conditions. We will assume that the only source of a fluctuation effect is an optimally tuned optical indicator. By equating the expression for the minimum detectable moment (3.25) to the moment of force in the experiment conducted by Doll and Näbauer (2.6), we obtain

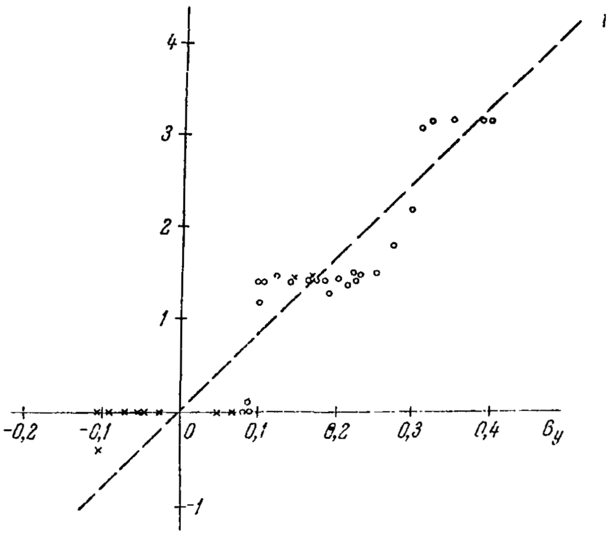

Figure 18

$$[\Phi_y]_{\min} \quad \zeta \frac{8\pi}{\hat{\tau} B_x l} \sqrt{\hbar \omega_{\text{mech}} I / A}, \tag{6.3}$$

where $\hat{\tau}$ is the duration of the sinusoidal train, inducing the superconducting cylinder with the moment of inertia I and the characteristic frequency ω_{mech} into resonance. If in (6.3) it is assumed that B_x = 10 oe, $l = 6 \cdot 10^{-2}$ cm, $I = 5 \cdot 10^{-11}$ g·cm^2 (data from the experiment by Doll and Näbauer), $\hat{\tau} = 10^3$ sec, ω_{mech} = 1 sec^{-1}, $A = 10^2$, $\zeta = 2$, then $[\Phi_y]_{min} = 5 \cdot 10^{-20}$ gauss·cm^2, i.e., almost 13 orders of magnitude less than $\Phi_0 = 2.06 \cdot 10^{-7}$ gauss·cm^2. Thus, there can be a more detailed checking of the discovered quantum macroscopic effect while in general employing the experimental method described above.

It should be noted in conclusion that quantum macroscopic effects are evidently not limited to the two cases considered above. In a certain sense, formulas (3.17) and (3.25) must also be regarded as quantum limitations in macroscopic experiments. We can note the existence of other quantum macroscopic effects, in particular, those associated with experiments with test bodies. For example, it is easy to see that a macroscopic mechanical oscillator to a certain degree will not interact during the time $\hat{\tau}$ with an optical indicator if the initial amplitude of its oscillations x_{in} is less than /76

$$x_0 \simeq c\sqrt{\frac{\hbar}{N_0 \nu_0 \tau}}, \tag{6.4}$$

where N_0 is the intensity of the flux incident on it, ν_0 is the mean frequency of optical radiation, c is the speed of light. When $x_{in} < x_0$ the light pressure fluctuations should impart through the oscillator an energy less than $\hbar\omega_{mech}$. Assuming that N_0 = 1 erg/sec, $\nu_0 = 5 \cdot 10^{14}$ sec^{-1}, $\hat{\tau} = 10^3$ sec, we obtain $x_0 \simeq 1 \cdot 10^{-12}$ cm.

§ 7. Search for Elementary Particles With a Fractional Electric Charge

As is well known, the unit of an electric charge is $e = 4.80298 \cdot 10^{-10}$ CGSE. The charge of any body can vary only discretely by this value. This fundamental circumstance, which became known after the classical experiments by Millikan, who measured the charge of an electron, has been repeatedly checked. In particular, the equality in absolute magnitude of the electron and proton charges has been checked [42]. The result of this experiment confirmed the equality of charges with a high accuracy.

We note that the discreteness and equidistance of electric charges, as well as the absence of a "finer charge substructure", is an empirical law which is not mandatory from the point of view of the law of conservation of charge

$$\left(\frac{\partial\rho}{\partial\tau} + \operatorname{div} j = 0\right).$$

A burst of interest appeared in this fundamental problem in connection with the hypothesis formulated by Gell-Mann and Zweig [43, 44] concerning the existence of superelementary particles, so-called quarks, which should have an electric charge fractional relative to e (for different varieties of quarks the charge should be ± 1/3 e and ± 2/3 e).

The Gell-Mann -- Zweig hypothesis found indirect confirmation in the discovery of Ω^--particles. According to this hypothesis, at least one of the "varieties" of quarks should be stable. On the basis of the "hot universe" theory it was found possible (Ya. B. Zel'dovich [45]) to obtain a mean estimate at the present time of the distribution of relict quarks. The distribution level should be about 10^{-10}-10^{-11} quark per nucleon. However, this estimate is an average for the universe, and it is not impossible that there are accumulations of relict quarks in individual regions of the universe or in substances with a certain composition. Thus, an urgent need arose for repeating the Millikan experiments, or experiments close to them, in order to detect rare stable particles with a charge of 1/3 e or 2/3 e. Such experiments were carried out by G. Gallinaro and G. Morpurgo [46] at the University of Genoa, and also at Moscow University (Ya. B. Zel'dovich, L. S. Korniyenko, V. K. Martynov, V. V. Migulin, S. S. Poloskov and V. B. Braginskiy [47-49]). /77

Below we will describe the experiments carried out at Moscow University, which are of interest in connection with the above-mentioned fundamental problem (and not in relation to the fundamental nature of the result) and also because these experiments can be regarded as an illustration of the development of a sensitive electrometer.

In the search for stable relict quarks in solid bodies it is possible to use a modification of the Millikan method. It was desirable to determine the minimum charge (less than the electron charge) for a test body whose mass was several orders of magnitude greater than the mass of droplets in the Millikan experiments, since in the latter the number of nucleons in a drop was 10^{12}-10^{13}. However, the increase in test body mass by several orders of magnitude did not make it possible to retain the Millikan method in pure form, since in order to hold the drops in the earth's gravity field, it was already necessary to have an electric field strength of about 5 kV/cm. Accordingly, it was necessary to suspend the test body either by using a servosystem (ferromagnetic body), or using a Braunbeck suspension [50] (strong diamagnetic), which Gallinaro, Morpurgo, et al. propose for use in searching for quarks. The test body, suspended in a magnetic potential well, will be displaced relative to the position of equilibrium if the potential /78 well is in an electric field and if the body is charged. This displacement, proportional to the charge, if it has a low absolute value, changes discretely with a change in the charge by one or a few electron charges. The presence of a quark in the body should have the following effect: in place of possible charges $(\ldots, -2, -1, +1, +2, \ldots)\, e$, one should observe charges $(\ldots, -2^1/_3, -1^1/_3, -^1/_3, +^2/_3, +1^2/_3, \ldots)\, e$, or $(\ldots, -1^2/_3, -^2/_3, +^1/_3, +1^1/_3, \ldots) e$.

Thus, instead of measuring the time of motion of droplets in the field of an electric capacitor, as was done by Millikan, it was necessary to investigate the distribution function of displacements of a test body relative to the position of equilibrium in the potential well. In addition, it was necessary to have sufficiently small displacements which would be linearly dependent on the charge magnitude.

Figure 19 shows /79
a schematic diagram of the central part of the apparatus used in making the measurements [47, 48]. The test body was a particle of graphite (diamagnetic) and the body was situated near the section between the flat poles of an electromagnet. Near the section, the field, decreasing sharply in the z direction, creates a force which is directed upward for the diamagnetic (graphite)

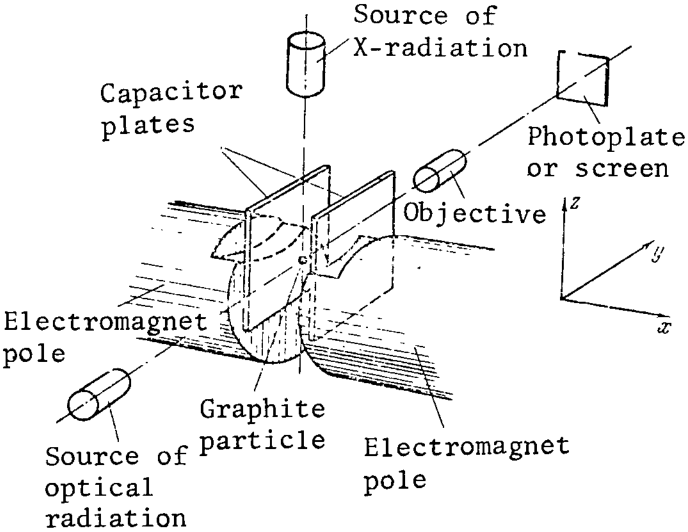

Figure 19

$$F_z = -V(1-\mu)(8\pi)^{-1}\frac{\partial H^2}{\partial z},$$

compensating the particle weight. The magnetic field strength near the section varied from $1.8 \cdot 10^4$ to $1.2 \cdot 10^4$ oe for different particles. The gap between the poles was about 5 mm. In the upper part of the electromagnetic poles there was a recess which ensured stability of the particle in the y direction. The particle is illuminated by a stabilized source of optical radiation. An objective makes it possible to obtain a persistent image of the particle on a screen (90-fold linear magnification).

In the gap between the electromagnet poles there are two well-insulated parallel plates of an electric capacitor. Thus, the direction of the electric field in the neighborhood of the particle coincides with the magnetic field direction. The equilibrium position of the particle under the joint influence of both fields and the small particle charges is linearly dependent on the charge magnitude q, assuming the same electric field strength E.

In the first series of measurements [47, 48], the volume around the particle was not evacuated and the plates were fed a constant electric voltage. In place of a screen, the authors used a photographic film and registered the difference in particle displacements with a change in voltage polarity. This excluded the possible effect of the force $V \frac{\partial E^2}{\partial x}$ (interaction of the induced dipole moment and a nonuniform electric field), changing quadratically with the field E. The particle charge q was varied using a weak current of ions in the air, obtained by exposure to the electric field and weak X-radiation (this same method was employed by Millikan).

Figure 20 (upper part) shows the distribution function for quasistatic /81
displacements x of a particle having the mass $9 \cdot 10^{-9}$ g in an electric field E = 1.5 kV/cm. There are six clearly visible equidistant maxima, the distance between which (about $5 \cdot 10^{-4}$ cm) corresponds to a change in the particle charge for e. The mean charge at the zero maximum is statistically indistinguishable from zero: $q = 0.00 \pm 0.11\ e$, with a reliability of 0.99. (The statistical processing of these measurements has been described in detail in [47].) Figure 20 shows that the particle charge varied from + 2 e to - 3 e. The scatter of particle displacements was caused by Brownian motion. Accordingly, the points corresponding to different charges are

denoted in the figure by different symbols (crosses, circles, etc.). The lower part of Figure 20 shows a record of this same particle in time, averaged for 8 displacements $\bar{x}_8$ (the time axis is directed downward). For $\bar{x}_8$ the Brownian fluctuations are expressed to a lesser degree and the deviations of displacements from the mean do not overlap. We can clearly see the discrete change in the position of equilibrium for the particle, as well as the temporal repetition of the same charges (+ 1 e and - 2 e). The letter R denotes the times when the X-radiation source was operative. In order to obtain such a distribution function it is necessary that $\tau_{meas} \approx 10^4$ sec.

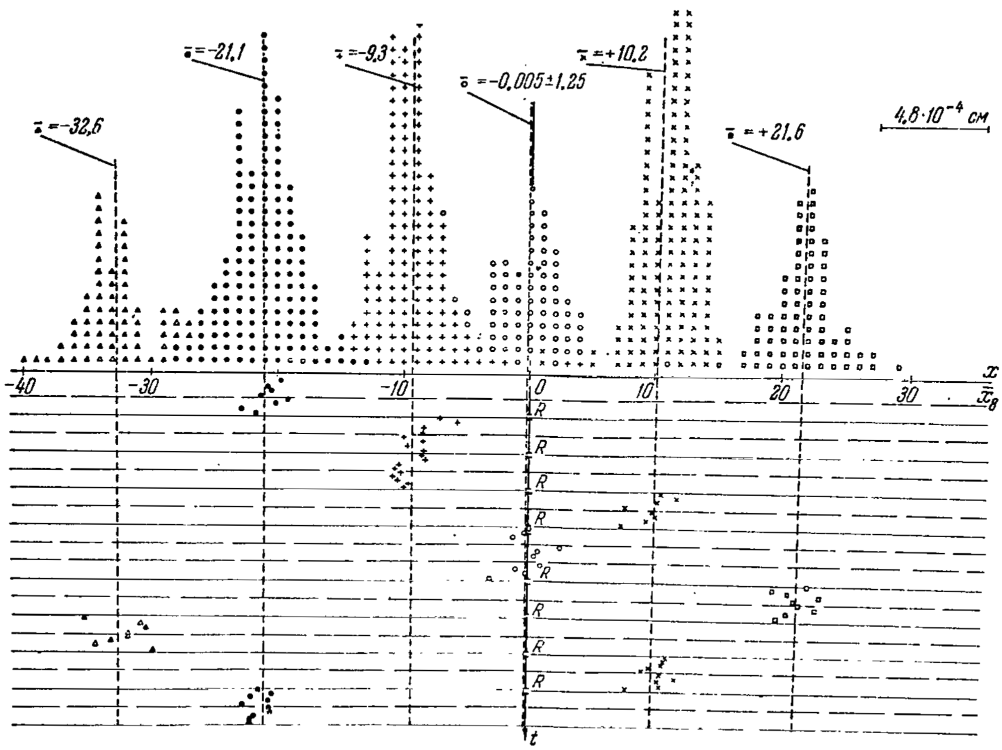

Figure 20

A possible source of systematic error in this method is that the particle may have a static dipole moment D, interacting with the nonuniform electric field ($\partial E_x/\partial x \neq 0$). This can result in the simulation of a fractional charge. It was found that some particles of pure graphite and graphite with added impurities have a dipole moment $D_x \sim 5 \cdot 10^{-9}$ CGSE (this corresponds to a potential difference of about 1 V on the particle surface). In order to eliminate this parasitic effect, the electric capacitor plates were carefully inserted parallel to one another (parallelism not worse than 7-8 μ at a distance of 10 cm); in addition, measures were taken for precluding the falling of graphite dust on the capacitor plates near the particle position of equilibrium (for further details see [47]).

In the first series of measurements (with the sums of masses of all particles taken into account), it was demonstrated that no quarks were present at the level of occurrence $1 \cdot 10^{-17}$ quark per nucleon. A solution of /82 a stony meteorite, as well as dry residue from the slow evaporation of a great quantity of water, was added to the graphite (in this procedure there might be an "enrichment" with quarks and accordingly the estimated limit for occurrence of quarks in water was about 10^{-22} quark per nucleon).

In the described apparatus, as mentioned above, the time of measurement with one particle was $\tau_{meas} \simeq 1 \cdot 10^4$ sec; the relaxation time due to the presence of air was $\tau^* \simeq 1$ sec. Thus, the conditions for measuring the force $F_x = \frac{e}{3} E$, whose effect on a particle was to be determined, were extremely far from optimum. In the second series of measurements [49], changes were made in the apparatus. The volume near the particle was evacuated (vacuum about $1 \cdot 10^{-2}$ mm Hg). As a result, the quality of the particle for oscillations in the direction of the electric lines of force attained $Q \simeq 10^2$. With a fore-vacuum it was possible to change the particle charge by use of a weak glow discharge. Instead of a photographic film, a photoelectric converter was placed in the screen plane; the signal at the output of this converter was proportional to the particle displacement. Due to the relatively high quality of particle oscillations it was convenient to replace the quasistatic electric voltage across the capacitor plates by a variable electric voltage with a frequency equal to the frequency of characteristic oscillations of the particle in the x direction ($\omega_x \simeq 2\pi \cdot 7$ sec^{-1}). The particle oscillations were registered on the tape of a loop oscillograph, and then the record was subjected to statistical processing, similar to the synchronous rectification

operation. This made it possible to retain the signal-to-noise ratio the same as in the first series (confidence interval about 0.1 e), increasing the particle mass on the average by a factor of 7, and reducing the measurement time with one particle to 10^2 sec. Salts of a solution of marine concretions in fluoric acid, or the precipitate from evaporation of a great quantity of water, were added to some of the 30 particles (mean mass about $1.2 \cdot 10^{-7}$ g) which were measured, as in the first series. In not one of these particles was it possible to detect the presence of fractional charges. The /83
charge which was minimum in absolute value did not differ statistically from zero, and the mean confidence interval when measuring the charge was 0.093 e (at the confidence level 0.99).

Thus, the second series of measurements made it possible to reduce the upper limit of possible occurrence of relict quarks in solid bodies to 10^{-18} per nucleon (taking into account possible enrichment due to the evaporation of water to 10^{-23} per nucleon).

In the experiments by Gallinaro, Morpurgo, et al. [46], a negative result was also obtained. The method in this experiment was similar to that described above. However, the capacitor plates were arranged (see Figure 19) in such a way that the electric field E was directed along the y axis. This made possible a substantial increase in the particle displacement with a change in q by one electron, since the rigidity in the potential well in the y direction was less than in the x direction. However, the lesser size of the capacitor plates, as a result of such an orientation, considerably increases the possible $\partial E_y/\partial y$ value, and therefore, due to the above-mentioned parasitic effect sets a more rigorous limit for particle mass. In this experiment the particle mass was two orders of magnitude less than in the second series of our measurements.

Both described experiments were of a purposeful nature: an attempt was made to detect the existence of rare relict quarks in a solid body. Accordingly, it was desirable, while retaining a resolution of about 0.1 e (confidence interval), to have a test body of the greatest possible mass. It is clear that other experimental variants can be carried out. For example, one could attempt to find rare stable particles with a charge such as $1 \cdot 10^{-3}$ e

using a similar apparatus. This would require a decrease in the mass of the test bodies by approximately three orders of magnitude. However, no theoretical premises, such as the Gell-Mann — Zweig hypothesis, yet exist for such a search.

In § 4 we presented an estimate of the minimum charge $[q]_{min}$ which can be detected in a body with the mass m when it is acted upon by a force qE under optimum conditions. For $E = 10^2$ CGSE (30 kV/cm), $m = 1$ g, $\hat{\tau} = 1 \cdot 10^3$ sec and $\omega_{mech} = 1$ sec^{-1} a value $[q]_{min} = 1.6 \cdot 10^{17}$ CGSE $= 3 \cdot 10^{-8}$ e was obtained. /84
Thus, in these experiments with test bodies there is a great reserve of sensitivity.

We note in conclusion that a certain caution must evidently be exercised with respect to the lower limit of possible occurrence of quarks obtained in [51], since the authors of [51] used an "enrichment method" which with certain assumptions concerning the properties of quarks could lead to an impoverishment of the substances subjected to investigation.

CHAPTER III

/85

PROSPECTS FOR CARRYING OUT GRAVITATIONAL AND NUCLEAR EXPERIMENTS WITH TEST BODIES

Relativistic gravitational experiments occupy a special place in experimental physics. The unusual difficulty in carrying them out, together with the smallness of the gravitational constant, on the one hand, and the real need for a theory in conducting these experiments are attracting the attention of many researchers, and this has led to numerous proposals for formulating experiments. The relatively vigorous development of relativistic gravitational theory, especially during the last decade, has led to a situation which is anamolous in comparison with other branches of physics: researchers can not "contend" with the effects which were essentially predicted more than 40 years ago. There is a certain justification for this situation because for those masses which the experimenter has at his disposal in the laboratory, the effects predicted by the general theory of relativity are unusually small, and as will be demonstrated hereafter, many of them can be detected only near the threshold of limiting sensitivity, from which experimenters today are separated by 7 to 10 orders of magnitude. Another possible factor is that at first glance the effects predicted by the general theory of relativity have no practical application because they lead only to small additions to the non-relativistic Newtonian description of the motion of artificial space bodies, and photon rockets are still largely in the realm of science fiction /86 writers. The recently appearing investigations relating the structure of elementary particles and masses collapsing to the Planck elementary length (planckeons-maximons [52]) in general do not exclude the possibility of existence of great energies of gravitational origin in elementary particles.

We now know of the results of three fundamental experiments which confirm the general theory of relativity. These include the rotations of the perihelion of Mercury, the red-blue shift in the frequency of electromagnetic radiation in the earth's gravity field, and the deflection of the optical radiation of stars in the sun's gravity field. It should be noted that in the third experiment it was not possible to obtain a high measurement accuracy and accordingly it was impossible to obtain a good correspondence between the results and the predictions which follow from the general theory of relativity (see review [53]).

In Chapter III we will discuss the possibilities of detecting some new relativistic gravitational effects involving experiments with test bodies (§§ 8 and 9). This chapter will also include a brief discussion of two nuclear experiments for the detection of small forces and the moments of forces applied to macroscopic bodies.

§ 8. Problem of Detecting Gravitational Radiation.

It has been known for more than 40 years that in the case of a weak gravitational field the Einstein equations are similar to the wave equations for the electromagnetic field [54]

$$\Box \Phi_{\mu}^{\nu} = -\frac{16\pi\gamma}{c^4} T_{\mu}^{\nu} \qquad (8.1)$$

with the additional condition $\Phi_{\mu,\nu}^{\nu} = 0$. In equation (8.1)

$$\Phi_{\mu}^{\nu} = h_{\mu}^{\nu} - \frac{1}{2}\delta_{\mu}^{\nu} h_{\alpha}^{\alpha},$$

where h_{μ}^{ν} is a small value of the first order of magnitude characterizing the curvature of the metrics of space $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$, T_{μ}^{ν} is the energy- /87 momentum tensor; γ is the gravitational constant; c is the speed of light. These equations, as in the Maxwellian equations, have a solution in the form of waves propagating with the same velocity as electromagnetic waves. However, only during recent years has there been discussion in the literature concerning the possibility of detecting gravitational radiation. It is difficult to detect gravitational waves: a) due to the smallness of the gravitational constant γ, and b) because the ratio of the gravitational mass (gravitational charge) to an inert mass is a constant value for any bodies (see § 5). Due to the second circumstance, the variable motion of masses can lead only to quadrupole radiation of gravitational waves. For example, the energy loss in gravitational radiation by some system of masses when $v/c \ll 1$ is equal to [54]

$$\frac{d\mathscr{E}}{d\tau} = -\frac{\gamma}{45c^5}(\dddot{D}_{\alpha\beta})^2, \tag{8.2}$$

where $D_{\alpha\beta}$ is a component of the tensor of quadrupole moment of masses

$$D_{\alpha\beta} = \int_V \mu (3x^\alpha x^\beta - \delta_{\alpha\beta} x_\gamma^2)\, dV. \tag{8.3}$$

Here μ is density, V is volume. Expression (8.2) for the intensity of gravitational radiation with an accuracy to the numerical factor coincides with the similar expression in electrodynamics for the intensity of quadrupole electromagnetic radiation

$$\frac{d\mathscr{E}}{d\tau} = \frac{1}{180c^5}(\dddot{D}_{\alpha\beta})^2, \tag{8.4}$$

if in this expression $\sqrt{\gamma}\ \mu$ is replaced by the density ρ of electric charges.

Thus, in an attempt to carry out experiments for detecting gravitational radiation the situation is approximately the same as in electrodynamics, but the experimenter will have at his disposal only gravitational charges (gravitational masses) of one sign with the same ratio of the gravitational charge /88 (gravitational mass) to the inert mass. Accordingly, possible forces and detectors can only be of the quadrupole type, and therefore extremely ineffective. In addition, the specific gravitational charge is extremely small (for an electron $\sqrt{\gamma} m_{grav}/m_{in} = \sqrt{\gamma}$ is less than e/m_{in} by $\sim 10^{21}$). Gravitational radiation is visualized as a field detached from nonuniformly moving masses; this field decreases with distance from the source as r^{-1}, provided the distance r is much greater than the wavelength. In other words, as in the case of electromagnetic radiation, here we can discriminate a wave zone in which the change in the metrics of space is propagated with a velocity equal to the speed of light and decreases as r^{-1}, and a nonwave zone in which the field can be computed approximately using Newton's law.

In addition to the continuous improvement in experimental equipment, an objective basis for the new interest in carrying out experiments for detecting gravitational radiation is evidently the relatively recent development of statistical methods for discriminating a weak signal from noise with the optimum use of preliminary information on the signal. This section gives data on different possible forces of gravitational radiation, discusses the attainable response of detectors on the basis of expressions derived in Chapter II, and gives the preliminary results of some experiments which make it possible to estimate the upper limit of the level of gravitational radiation of extraterrestrial origin.

Sources of Gravitational Radiation

Binaries. Binary stars with a small period of rotation are the most reliable sources of gravitational radiation of extraterrestrial origin. These forces are virtually constant for the terrestrial observer, and therefore in an attempt to detect their radiation it is possible to achieve a prolonged discrimination of the signal from the noise during correlated reception, taking advantage of the circumstance that the radiation is rigorously synchronous with the rotation of the binary components, which can be optically /89 observed. In the case of a system of two stars moving in circular orbits and having the masses m_1 and m_2 and the rotation frequency ω, the intensity of gravitational radiation can be computed using formula (8.2); for this case the formula assumes the form

$$\frac{d\mathcal{E}}{d\tau} = -\frac{32}{5}\frac{\gamma^{7/3} m_1^2 m_2^2 \omega^{10/3}}{(m_1+m_2)^{2/3}}. \tag{8.5}$$

As can be seen from (8.5), binary stars with a great mass, small period of rotation, and situated relatively close to the terrestrial observer are of interest as intensive forces of gravitational radiation. Table 4 gives data on the intensity of gravitational radiation for six binaries situated relatively close to the solar system. This same table gives the period of rotation τ_{rot}, the masses m_1 and m_2 in units of solar mass, the distance L to these stars from the earth, and the density t of the flux of gravitational radiation near the earth. The factor A can vary from 0 to several units, depending on the orientation of the rotation plane of these stars relative to the earth. The first five stars in this table are eclipsing binaries (see the Kopal catalogue [55]), whereas the last star WZ in the constellation Sagitta has a uniquely small period of rotation (81 minutes). The total intensity flux of gravitational radiation from this star possibly exceeds the intensity of optical radiation [56]. /90

TABLE 4

Star	τ_{rot}, Days	m_1	m_2	L, cm	$dE/d\tau$, erg/sec	$A \cdot t$, erg/sec·cm	$\frac{\Delta\omega}{\omega}\Big\vert_{\tau = 3\cdot10^8}$ sec
UV Leo	0,69	1,[illegible]6	1,25	$2{,}1\cdot10^{10}$	$1{,}8\cdot10^{31}$	$3{,}5\cdot10^{-12}$	$3{,}2\cdot10^{10}$
V Pup	1,45	1,66	9,8	$1{,}2\cdot10^{21}$	$4\cdot10^{31}$	$2{,}3\cdot10^{-12}$	$1{,}4\cdot10^{10}$
i Boo	0,268	1, 5	0,68	$3{,}8\cdot10^{10}$	$1{,}1\cdot10^{30}$	$1{,}1\cdot10^{-10}$	$1{,}9\cdot10^{9}$
YY Eri	0,321	0,76	0,50	$1{,}3\cdot10^{10}$	$2{,}6\cdot10^{9}$	$1{,}3\cdot10^{1}$	$5{,}1\cdot10^{10}$
SW Lac	0,321	0,97	0,8[illegible]	$2{,}3\cdot10^{20}$	$1{,}1\cdot10^{30}$	$1{,}7\cdot10^{-1}$	$9{,}5\cdot10^{10}$
WZ Sge	81max	0,6	0,0[illegible]	$3\cdot10^{10}$	$3.5\cdot10^{29}$	$3\cdot10^{-13}$	$4\cdot10^{9}$

Note: Commas represent decimal points.

According to estimates made by V. N. Mironovskiy [57], binaries of the type WU Ma should yield the greater part of the flux density of gravitational radiation of nonterrestrial origin; this value should be approximately 10^{-9} erg/sec·cm^2. The most probable period of rotation of these stars is about 4 hours. Thus, if a gravitational radiation detector is created under terrestrial conditions and is intended for the reception of radiation from known binaries, it must be able to register at least intensity fluxes of $t \approx 10^{-9}$-10^{-10} erg/sec·cm^2. Such an intensity flux, if it were electromagnetic radiation, could be detected without difficulty. However, as will be shown below, the quadrupole nature of the detector makes this a very difficult undertaking.

The radiation of gravitational waves leads to an energy loss of the binary; as is known [54], this loss is equal to

$$\mathscr{E} \simeq -\gamma m_1 m_2 (2R)^{-1}$$

(without relativistic corrections). As a result, the components of the binary star should converge, and the frequency of rotation should increase. During the time $\hat{\tau}$ the frequency of rotation ω should change by the value $\Delta\omega$:

$$\frac{\Delta\omega}{\omega} \simeq \frac{96\gamma^{5/3} m_1 m_2 \omega^{8/3} \hat{\tau}}{5 (m_1 + m_2)^{1/3}}. \qquad (8.6)$$

Table 4 gives the relative change in the frequency of rotation $\Delta\omega/\omega$, computed using (8.6) for the same binary stars during the time $\hat{\tau} = 3 \cdot 10^8$ sec (about 10 years). The table shows that all six stars exhibit a change in the frequency of rotation; this can be caused by gravitational radiation greater than the relative frequency instability of modern atomic and molecular frequency standards (for a hydrogen maser about $2 \cdot 10^{-12}$). It is interesting to note that for most stars the rotation frequency is known with an accuracy to the 8th decimal place, whereas the effect of gravitational radiation is expressed, as is clear from the table, in the 9th or 10th place. Thus, a /91 possible indirect experiment which could confirm the existence of gravitational radiation would involve the long-term observation of change in the period of rotation of suitable binaries.

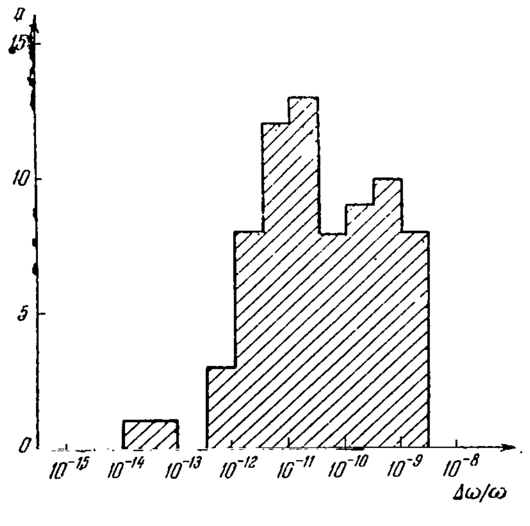

Figure 21

Other phenomena can lead to a change in the period of rotation (for example, expulsion of a large mass of matter from one of the binary components); this makes difficult the possibility of observing the effect in pure form.

The number of binary stars in which the change in period of rotation due to the radiation of gravitational

waves theoretically can be detected using modern frequency standards is rather great. Figure 21 shows the distribution of the number of known eclipsing binaries as a function of the postulated change in the frequency of rotation $\Delta\omega/\omega$ for $\hat{\tau} = 3 \cdot 10^8$ sec. In determining this distribution data were used for m_1, m_2 and τ_{rot} from the Kopal catalogue [56]. As can be seen from /92
the histogram, 8 eclipsing stars must change their frequency of rotation by more than $1 \cdot 10^{-9}$, whereas 48 should change by more than $1 \cdot 10^{-11}$.

We note one interesting circumstance: in the compilation of Table 4 binary stars were selected which had relatively large m and small τ_{rot}. It was found that among the known binaries there are none whose components have masses of several solar masses and a period of rotation equal to or less than the period of WZ Sge (81 minutes). If such binaries existed, as a result of energy loss in gravitational radiation their lifetime as binaries would be relatively short. In the case of the "unique" binary WZ Sge, the lifetime hypothetically can be about 100 million years. Thus, the absence among known binaries of those with large $\Delta\omega/\omega$ can be regarded as some indirect confirmation of the existence of gravitational radiation.

Hypothetical sources of gravitational radiation. The processes occurring during asymmetrical star collapse can lead to powerful gravitational radiation. As pointed out by Ya. B. Zel'dovich and I. D. Novikov [58], with the falling of a body with the mass m onto a spherically symmetrically compressing star with the mass M, whose radius is close to the gravitational radius $r_g = 2\gamma M/c^2$, several percent of the energy mc^2 is transformed into a burst of gravitational radiation, provided that $m \sim M$. The gravitational wave, during movement along the radius of a star with M, has the form of a single burst with the duration $\Delta\tau \sim r_g/c$; in the case of finite movement in an orbit with a radius comparable with r_g, it has the form of a train of such bursts. Each burst has an energy of $\sim \alpha m^2 c^2/M$, where $\alpha \simeq 0.01$-0.1. It is interesting that in the case of finite movement of the mass m in the neighborhood of the star M the total radiated energy in the form of gravitational waves is not dependent on the ratio m/M. If such a source was situated at a distance of $L = 500$ mps from the solar system, when $m = m_\odot$ and $M = 10^2\ m_\odot$ one could expect near the earth a flux with the intensity

$$t \simeq \beta \frac{mc^5}{4\pi L^2 r_g} \simeq 0.7 \frac{\text{erg}}{\text{sec} \cdot \text{cm}^2} \quad (8.7)$$

if it is assumed that $\beta = 10^{-2}$. In this case the greater part of the emission spectrum should lie near the frequency $f \simeq c/r_g = 10^3$ cps. However, it is unknown how frequently such processes transpire; therefore, if a detector was created for the reception of radiation from such sources, it would be necessary to plan on observations over a long period of time. /93

In the case of asymmetrical star collapse, intensive gravitational radiation can occur due to other mechanisms (star rotation and oscillation). According to estimates by I. S. Shklovskiy and N. S. Kardashev [59], made on the basis of some model concepts of asymmetrical star collapse, it can be expected that with a mass $M = 10^{41}$ g the intensity of gravitational radiation would attain 10^{54}-10^{58} erg/sec. If such a source was situated at a distance of 500 megaparsecs from the earth, a flux of gravitational radiation $t \simeq 10^{-1}$--10^{+3} erg/sec·cm^2 could be expected from it near the frequency $f \simeq 10^{-4}$ cps.

Ya. B. Zel'dovich [60] feels that it is not impossible that pulsars are also sources of gravitational radiation synchronous with the pulsation frequency.

If so-called neutron stars do exist [61], which should have relatively large masses ($m \sim 0.5\ m_\odot$) and small size ($R \sim 16$ km), double neutron stars should also be a source of powerful gravitational radiation. According to estimates made by Dyson [61], a double neutron star in the two seconds prior to the merging of its two components emits $\sim 10^{52}$ erg/sec with a frequency of about 10^3 cps. If such a source is situated at a distance of 300 kps from the earth, it can be computed that its flux intensity would be 10^3 erg/sec·cm^2.

Gravitational radiation from the above-mentioned sources is caused by the dissipation of energy, not by its transfer from one part of interacting masses to another [54]. We note that these hypothetical sources of gravitational radiation are not constant, and although they should give considerably greater flux densities of gravitational radiation over a brief period than known binaries, it is desirable in formulating a corresponding experiment to /94 estimate how frequently such sources may be active. Such estimates have not yet been made.

<u>High frequency gravitational radiation of nonterrestrial origin.</u> The thermal motion of matter may be a possible source of gravitational radiation. According to an estimate made by V. N. Mironovskiy [57], the intensity of solar gravitational radiation is caused for the most part by bremsstrahlung (gravitational radiation) during the Coulomb scattering of electrons and is about 10^{12} erg/sec. This flux intensity corresponds to several gravitons (with a frequency approximately corresponding to the optical range) incident on a square meter of the earth's surface per second. If we adhere to the hypothesis of possible mutual transformations of ordinary matter and the gravity field (D. D. Ivanenko, A. A. Sokolov [62]), we can estimate the effective cross section of reactions at which the gravitational transmutations of fermions occur [63, 64, 65]. However, the cross section of such reactions is extremely small: according to an estimate made by G. M. Gandel'man and V. S. Pinayev, the gravitational radiation during Coulomb scattering of electrons is 10 orders of magnitude less than the radiation of neutrinos [66]. Accordingly, real experiments with this source of transformations have evidently not yet been discussed.

We will mention still another possible mechanism which can give rise to high-frequency gravitational radiation. During the propagation of electromagnetic radiation in a constant electric or magnetic field varying in time with the frequency of the electromagnetic radiation, the components of the energy-momentum tensor, in accordance with equation (8.1), should yield gravitational radiation of the same frequency [67]. Since the propagation velocities for both waves are identical, there should be a wave resonance of gravitational and electromagnetic waves. In the absence of a constant field there is no radiation of gravitational waves. The transformation efficiency can be characterized by the ratio of the amplitudes of the gravitational wave $a(x)$ and of the electromagnetic wave $b(x)$. Assuming that plane waves are propagated in the direction x and interact during the time τ, it can be seen [67], that /95

$$\left|\frac{a(x)}{b(0)}\right|^2 \simeq \frac{\gamma}{\pi c^3}(P^0)^2\tau^2, \tag{8.8}$$

where P^0 is the strength of the permanent magnetic field. If it is assumed that $P^0 = 10^{-5}$ oe, $\tau = 3 \cdot 10^8$ sec and it is assumed that the total time for movement of electromagnetic radiation from a remote cosmic source is 10^7 years, then $|a(x)/b(0)|^2 \approx 10^{-17}$. It is important that in this case we are not involved with "reddening" of all the photons entering into the electromagnetic wave, but with the transformation of a photon into a graviton.

Possible terrestrial sources of gravitational radiation. Evidently, under terrestrial conditions it is difficult to create a source of gravitational radiation which could yield an intensity comparable with the intensities from extraterrestrial relatively low-frequency sources (nonstationary processes during star collapse and radiation of binary stars). For example, if a rod with a mass $m \sim 10^4$ g is rotated at such a velocity that the centripetal stress in it is close to the ultimate strength of the best varieties of steels, the maximum intensity of gravitational radiation which can be obtained with the corresponding form of rod is 10^{-30} erg/sec (about 10 gravitons per year).

Mechanical oscillations in solid bodies also lead to gravitational radiation (Weber [68]). If longitudinal oscillations are excited in a rod at the lowest of its characteristic frequencies, the intensity of gravitational radiation can be computed using the formula

$$\frac{d\mathscr{E}}{d\tau} = -\frac{16}{15}\frac{\gamma\mu^2 S^2 \xi^2 v^6}{c^5}, \tag{8.9}$$

which after simple transformations can be obtained from (8.2) (see [68]). In formula (8.9) μ is the density of matter in the rod, S is its cross section, ξ is the amplitude of linear expansion, v is the velocity of propagation of longitudinal waves in the rod. According to Weber's estimates, under the best conditions for μ, S, ξ and v one can count on a flux with the intensity 10^{-13} erg/sec; however, in this case it would be necessary to expend a power of about 10^8 W on the excitation of mechanical oscillations. /96

In explosions it is also possible to expect a burst of gravitational radiation. Schuking (see the table in the book by Wheeler [69]) estimated that during the explosion of a uranium bomb (17 kilotons) the radiation intensity is 10^{-4} erg/sec for a period of about 10^{-8} sec.

In summarizing these estimates for different possible types of sources of gravitational radiation, in formulating corresponding experiments it evidently is necessary to give preference to nonterrestrial forces.

We should again mention the proposals made by U. Kh. Kopvillem, et al., [70] that the collective oscillations of molecules with a high mass quadrupole moment, excited by synchronous electromagnetic radiation from a powerful laser, be used for the radiation of gravitational waves (with a frequency corresponding to the optical range). It is proposed that phonon counters for these same molecules be used as a detector. The preliminary computations made by U. Kh. Kopvillem, et al., show that with this approach as well there are extremely great difficulties in implementing an experiment. Since these proposals in their physical nature are beyond the scope of this book, we will not discuss them in detail, but instead refer the reader to the literature.

Gravitational Radiation Quadrupole Detector

As already mentioned above, the gravitational radiation detector, like the source, must be of the quadrupole type. In other words, in formulating an experiment for the reception of gravitational radiation it is necessary to have at least two test masses. Since their specific gravitational charges are identical (an identical ratio of gravitational to inert mass), the relative movement of test masses will be caused only by the wave gradient. If the distance between masses l^{α} is small in comparison with the wavelength, and their velocities are not too great ($v/c \ll 1$), the difference in forces acting on the two test masses in the gravitational wave field, according to Weber [68], is /97

$$F^{\mu}_{gr} \simeq - mc^2 R^{\mu}_{0\alpha_0} l^{\alpha}. \tag{8.10}$$

In formula (8.10) m is the magnitude of each of the test masses and $R^{\mu}_{0\alpha_0}$ are the components of the Riemann curvature tensor. In electrodynamics the difference in the forces acting on two identical electric charges q spaced at the distance l is

$$F_{el} \simeq q \frac{\partial E}{\partial l} l. \tag{8.11}$$

Expressions (8.10) and (8.11) are similar; the parameter $R^{\mu}_{0\alpha_0}$ is equivalent to the field strength gradient.

If the registry of a sinusoidal electromagnetic wave

$$E = E_0 \sin(\omega_0 \tau - Kx)$$

is accomplished using two identical electric charges (with an identical ratio q/m_{in}), from the difference in the forces F_{el} acting on these charges, knowing the frequency ω_0, the magnitude of the charge q and the distance between charges l, it is also easy to compute the intensity of the electromagnetic radiation passing near these charges. In the expression for the Poynting vector $S = c(4\pi)^{-1} \times [EH]$ it is necessary to substitute F_{el} from (8.11) and use $E = E_0 \sin(\omega_0 \tau - kx)$. Then, for the intensity of the sinusoidal electromagnetic wave we obtain

$$|S| \simeq \frac{c^3 F^2_{el}}{4\pi q^2 \omega_0^2 l^2}. \tag{8.12}$$

Formula (8.12) is correct if $l \ll \lambda$ and the velocities of charge motion are small.

If similar calculations are made for the intensity t of gravitational radiation, a similar expression can be derived

$$|t| \simeq \frac{c^3 (F^{\mu}_{gr})^2}{8\pi\gamma m^2 l^2 \omega_0^2}. \tag{8.13}$$

Formula (8.13), like (8.12), was derived for a sinusoidal wave. As can be seen from a comparison of formulas (8.12) and (8.13), they, like the formulas for radiation intensity, coincide (with an accuracy to the numerical factor), if in (8.13) $m\sqrt{\gamma}$ is replaced by q. /98

Thus, gravitational radiation can be "received" by any pair of test masses if there is some device making it possible to register the small difference in forces acting on these test masses present in the gravitational wave field. As the pairs of such masses one can select: earth-satellite, earth-star, two planets, two test masses in the laboratory and an extended solid body in which the gravitational wave excites mechanical oscillations. Such a detector, like the electric quadrupole, has its directional diagram [68].

The formulation of an experiment for the reception of gravitational radiation must evidently be discussed from two points of view. First, it is necessary to determine the conditions under which the test masses must be placed if the experiment is carried out in the laboratory. Second, it is necessary to select a method for measuring small relative displacements between the test masses caused by F^{μ}_{gr} ("instrumental" limitations).

Now we will consider the determination of conditions for formulating an experiment. If there is an oscillator consisting of two identical point masses m which are connected to one another by a rigidity element K and an element with friction H_{mech}, the equation for the relative motion of these masses under the influence of gravitational radiation will have the form

$$m\frac{d^2\xi^\mu}{d\tau^2} + H_{\mathrm{mech}}\frac{d\xi^\mu}{d\tau} + K\xi^\mu = F^\mu_{\mathrm{gr}} + F^\mu_{\mathrm{fl}}, \tag{8.14}$$

where F^{μ}_{fl} is the sum of all fluctuation forces acting on the test masses in the direction μ.

In a case when the test masses are acted upon only by fluctuations of an indicator with optimum tuning in accordance with the criteria set forth in § 4, it is possible to obtain analytical expressions for the values: $[R^{\mu}_{0\alpha 0}]_{min}$ and $[t]_{min}$. In the case of an optical indicator, the absolute /99
minimum detectable component of the Riemann curvature tensor is

$$|[R]_{\mathrm{min}}| \simeq \zeta\frac{2}{\tau c^2 l}\sqrt{\frac{\hbar\omega_{\mathrm{mech}}A}{m}}, \tag{8.15}$$

where l is the distance between the masses, ζ is a factor of the order of several units, dependent on the selected confidence limit of detection, $\omega^2_{mech} = K/m$, A is a factor characterizing the statistics of fluctuations in the indicator (for independently emitting photons $A = 1$). Expression (8.15) is correct for a gravitational field sinusoidally changing with time (R is the amplitude of change in one of the components) with the frequency ω_{mech} and a train duration $\hat{\tau}$; in this case $\hat{\tau} \gg 1/\omega_{mech}$. Expression (8.15) follows directly from (3.17) if in (3.17) $[F_0]_{min}$ is replaced by (8.10). In accordance with the derivation of (3.17) (see §§ 3 and 4) the right-hand side of (8.15) will be twice as great if the gravitational field changes in impulses and the duration of the impulse $\hat{\tau}$ conforms to the condition $\hat{\tau} \ll 1/\omega_{mech}$.

By transforming from components of the curvature tensor to the flux density for gravitational radiation [57], we obtain the expression for $[t]_{min}$:

$$[t]_{min} \simeq \frac{c^3 \hbar}{2\pi\gamma\omega_{mech}} \cdot \frac{\zeta^2}{\hat{\tau}^2 m l^2}. \tag{8.16}$$

Expression (8.16) is correct for a sinusoidal wave. If the gravitational wave has the form of a short impulse ($\hat{\tau} \ll 1/\omega_{mech}$), the right-hand side of (8.16) is four times larger. Thus, (8.16) makes it possible to estimate the scales of the experiment necessary for attaining the necessary response.

Substituting into (8.16) $\hat{\tau} = 10^6$ sec, $m = 2 \cdot 10^5$ g, $l = 10^4$ cm, $\omega_{mech} = 10^{-3}$ sec (which approximately corresponds to the period of rotation of intensively emitting binary stars, see above), $\zeta = 2$, we obtain $[t]_{0.95} \simeq 1.5 \cdot 10^{-11}$ erg/sec·cm^2. This value is approximately an order of magnitude less than the radiation intensity for the binary star i Bootes which has the "best" intensity (see Table 4).

Thus, we can draw the hypothetical conclusion that from the point of view of the theoretically attainable response of a quadrupole detector, consisting of a pair of test masses and an optimum indicator, the gravitational radiation of close binaries can be detected. However, the cited numerical estimates /100 show how close the limiting response is to the necessary level even in the case of relatively large-scale experiments, and accordingly, how difficult such an experiment is. Evidently, in terrestrial laboratories it is impossible due to the high level of additional fluctuating effects. It is clear that it is easier to attempt to discover the more intensive radiation at higher frequencies from the hypothetical sources mentioned above, provided that the bursts from them are sufficiently frequent.

If two very distant satellites are used as test bodies, the theoretically possible response will be much greater (see expression (8.16)) and the attainable response will be determined only by the current level of experimental measurement techniques for small relative movements at great distances.

As an illustration of the experimental possibilities, we will examine still another variant of such an experiment. Let us visualize that as test masses we employ two heliocentric space stations separated by the distance l = 100 million km, and that a gravitational wave periodically changes the distance between them.

It is clear from what has been said above that if the mass of the stations is about $m \simeq 10^5$ g, $\omega = 10^{-3}$ sec^{-1} and $\hat{\tau} = 10^6$ sec, the threshold response for such a detector will be substantially lower, $t \approx 1 \cdot 10^{-10}$ erg/sec/cm^2 (the flux density in the neighborhood of the solar system from the star *i Bootes*, see Table 4). Accordingly, we will be concerned only with the possibility of measuring small periodic velocities at such great distances, and also the fluctuation effects on the space stations.

The amplitude of the periodic component of relative velocity Δv of the two stations, caused by the gravitational wave, is

$$\Delta v = l \sqrt{\frac{8\pi\gamma}{c^3} t}. \tag{8.17}$$

This simple expression follows from (8.14), provided that the stations are regarded as free masses. The latter is correct when the frequency of gravitational radiation is much greater than the frequency of revolution in orbit. /101

In the case of two heliocentric stations at a distance of 100 million km, the gravitational radiation of the star i Bootes, in accordance with (8.17), creates an amplitude of periodic change in relative velocity of $\Delta v \simeq 2.5 \cdot 10^{-11}$ cm/sec. The measurement of such relative velocities is not difficult in the laboratory for two closely spaced bodies. It is a substantially more complex problem to measure the periodic components of relative velocities for bodies separated by great distances.

We note that the already available method for measuring the velocities of satellites at such distances makes it possible to measure the relative velocities with a metrologic (absolute) accuracy of about 0.1 cm/sec (for example, see data on Mariner IV [71]). Since the accuracy in measuring the relative amplitude of a narrow-band signal (such as that of the gravitational radiation of binary stars) is usually 6 or 7 orders of magnitude greater than the accuracy in absolute measurements of this same parameter, even now, with the already attained resolution, it would be possible to measure fluxes of gravitational radiation at the level $t \simeq 10^{-2} - 10^{-4}$ erg/sec$\cdot$cm^2. Evidently, there is no basis for assuming the attained accuracy in relative velocity measurements to be the limit.

In summarizing these estimates, the hypothetical conclusion can be drawn that in space there is real hope for obtaining an adequate response of quadrupole detectors for discovering gravitational radiation from extra-terrestrial sources. The problem of how great a fluctuation effect is exerted by the solar wind and the magnetic field in the solar system on test bodies requires additional experimental investigations. However, using already available information on the physical properties of interplanetary plasma, some preliminary estimates can be made. The amplitude of relative acceleration of two heliocentric stations in a wave gravitational field of a star, in accordance with the cited computations, is $2.5 \cdot 10^{-14}$ cm/sec^2. If we use the /102
data on the solar wind cited in the review, we can compute the acceleration imparted to the station by the flow of solar plasma at distances of about 200 million km from the sun. If the "cross section" of the station is about 10^4 cm^2, and its mass is $3 \cdot 10^5$ g, the acceleration falls in the range

10^{-12}--10^{-13} cm/sec^2. Unfortunately, data on the spectral density of solar wind fluctuations near a frequency $\omega \approx 10^{-3}$ sec^{-1} are presently unavailable.

We note in conclusion that massive planetary satellites also can create variable accelerations (in the nonwave zone) of heliocentric stations with a period equal to the period of satellite revolution about the planet. If the satellites of Mars have a mass of about 10^9--10^{10} g, at a distance of 100 million km from Mars a heliocentric station will experience accelerations of approximately identical amplitude caused by the satellites and the radiation of i Bootes. However, with respect to the discovery of gravitation radiation, this circumstance will not be important because the interference is determined by phase and frequency.

Search for Gravitational Radiation of Extraterrestrial Origin

One of the few experimental teams working on the detection of gravitational radiation is a group under Professor J. Weber (University of Maryland). During 1959-1961, Professor Weber [68] (also see review [72]) made a detailed analysis of the possibility of laboratory construction of a model of a transmitter and detector of gravitational radiation based on the mechanical oscillations of extended masses. His computations revealed that the use of mechanical oscillations of extended masses leads to excessively large experimental scales (large transmitter and detector masses, high powers necessary for exciting oscillations in the transmitter, extremely long period for discrimination of signal from noise). This team is now engaged in intensive efforts to detect gravitational radiation of extraterrestrial origin from /103 some possible hypothetical sources which in theory can give a considerably greater flux density of gravitational radiation near the earth than a laboratory source having any reasonable size.

In the first variant of the detector [73] developed by the team led by Professor Weber, the extended body employed was an aluminum cylinder about 150 cm in length, about 60 cm in diameter, and with a mass of about 1.5 ton. This cylinder (Figure 22) was suspended on thin filaments to a frame consisting of steel blocks interlaid with rubber spacers (antiseismic filter). The cylinder and frame were placed in a vacuum chamber and the entire apparatus was placed outside the city limits, far from industrial interference. Only the lowest-frequency quadrupole type of cylinder oscillations is used in detecting gravitational radiation. Its frequency is $\omega_0 \simeq 10^4$ rad/sec, and its quality is $Q \simeq 10^5$; accordingly, from the entire possible spectrum of gravitational waves the apparatus "cuts out" only a relatively narrow frequency band $\Delta\omega \simeq 0.1$ rad/sec near $\omega_0 \simeq 10^4$ rad/sec, provided the time for discriminating the order of the relaxation time for this type of oscillations is about 30 seconds.

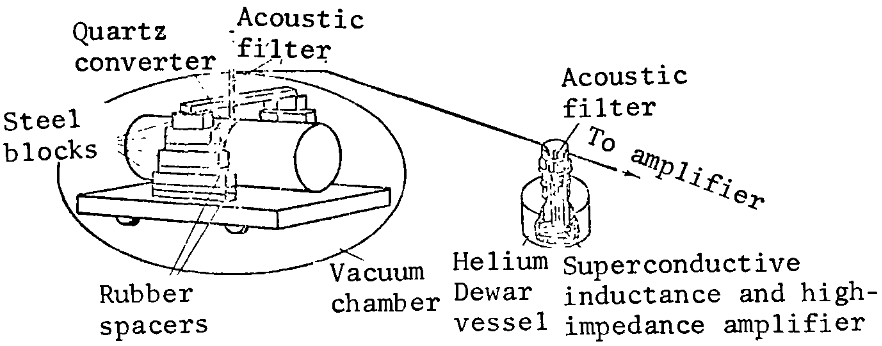

Figure 22

Quartz piezoelectric transducers, glued on the cylinder surface, make it possible to convert the mechanical oscillations of the cylinder into an electric signal. In removing the electric signal from the sensors, the matching problem arises; this was rather complex: the impedance of the quartz piezoelectric transducers glued to the cylinder was relatively high (about 10^9 ohms). In order to solve the matching problem it was necessary to use a superconductive inductance in the resonance preamplifier. As a result, the response of the gravitational detector was limited only by the Brownian oscillations of the aluminum cylinder. This means, for example, that the minimum detectable amplitude of oscillations of the cylinder ends (during a time approximately equal to the relaxation time) can be estimated from the condition /104

$$m\omega_0^2\overline{\delta^2} \gtrsim \varkappa T. \qquad (8.18)$$

at room temperature $\sqrt{\overline{\delta^2}} \gtrsim 2 \cdot 10^{-14}$ cm, which with a cylinder length of 150 cm corresponds to relative length changes (strain) of about 10^{-16}. We note that such a device is in theory an instrument for measuring mechanical stresses, not displacements.

The calibration of the gravitational detector was accomplished both using a standard noise source and directly by means of a dynamic gravitational field. The latter calibration variant, carried out by Sinsky and Weber [74] is essentially a high-frequency variant of the Cavendish experiment. The dynamic gravitational field was created by the oscillations of a second aluminum cylinder of somewhat smaller size at a distance of about 2 meters from the main cylinder. The output power of the detector approximately corresponded to the computed power, but the accuracy of such calibration was low.

Both calibration methods revealed that the response corresponding to the minimum detectable strain, computed using (8.18), was attained. The equivalent "gravitational" response can be determined using the expressions given by Weber in [73], which relate the strains appearing in an elastic body with the Riemann tensor component R_{i0j0}, causing acceleration of different parts of the test body relative to one another. In a case when the gravitational field changes sinusoidally in time with a frequency coinciding with the frequency of the lowest-frequency type of cylinder oscillations, and the /105
cylinder is oriented in the best possible way relative to R_{i0j0}, the relative change in cylinder length is

$$\varepsilon \simeq \frac{2c^2 Q}{\omega_0^2 \tau} R_{i0j0}, \qquad (8.19)$$

where c is the velocity of light propagation, Q is the quality of the type of oscillations. Substituting into (8.19) $\varepsilon = 10^{-16}$, $\omega_0 = 10^4$ rad/sec, $Q \doteq 10^5$, we obtain $R_{i0j0} \simeq 2 \cdot 10^{-34}$ cm^{-2}. This value corresponds to a flux density of gravitational radiation $t \simeq 2 \cdot 10^4$ erg/sec·cm^2.

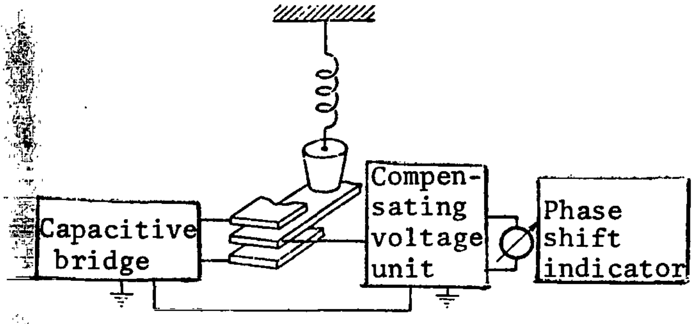

Figure 23.

A second variant of a gravitational detector developed by the group under Professor J. Weber is based on the idea of using the earth as an extended body. On the one hand, this appears extremely effective, since the cross section of radiation absorption is proportional to the detector mass; on the other hand, such a variant excludes, under terrestrial conditions, the possibility of using a coincidences circuit. The most low-frequency quadrupole type of terrestrial oscillations has a period of about 54 minutes and a quality about 400. A highly sensitive gravimeter [75] (Figure 23) was created at this frequency; it made it possible to register variations in the acceleration of gravity g exceeding the level $\Delta g/g \simeq 10^{-11}$. The results of study of the earth's noise background during the quietest period (in seismic respects) for the spectral /106 density of accelerations near the frequency $\omega \simeq 10^{-3}$ rad/sec gave $\overline{[\Delta g(\omega)]^2} \simeq 6.9 \cdot 10^{-14}$ gal$^2\cdot$sec/rad [75]. Comparison of this value with the mathematical expression [75] relating $\overline{[\Delta g(\omega)]^2}$ with the spectral density of the Riemann tensor enabled Weber to decrease somewhat the estimate for the upper boundary of the cosmic background of gravitational radiation (in the region of frequencies $\omega \simeq 10^{-3}$ rad/sec): $\overline{[R(\omega)]^2} < 6 \cdot 10^{-79}$ cm$^{-4}\cdot$rad$^{-1}\cdot$sec; the earlier estimate [68] was 3 or 4 orders of magnitude greater. The value of this estimate is relative because its corresponding energy density near the considered frequency must be $t \simeq 10$ erg/sec$\cdot$cm^2, whereas binaries in this same frequency range create a density of gravitational radiation $t \simeq 10^{-9}$--10^{-11} erg/sec$\cdot$cm^2 (see above).

This variant of a detector of the gravimetric type will undoubtedly become more promising if it can be used in a coincidence circuit, for example, having one detector each on the earth and on the moon, as is planned by the Maryland group in the next few years [75]. At the present time an experiment with a coincidence circuit with detectors of the first type is being conducted by Weber.

The idea of a coincidence circuit [76] involves the simultaneous use of two detectors which are separated by some distance. This method makes it possible to discriminate "gravitational bursts" against a background of internal fluctuations. In actuality, in this circuit gravitational radiation leads to correlated readings at the outputs of both systems (due to the large c the wavelength is also large), whereas internal thermal fluctuations cannot have such a correlation.

Detectors of the first type, placed in concrete chambers at a distance 10^3 km apart [76], were used in the experiment. One detector had the parameters described above; the other was smaller (the length was the same, but the diameter was about 20 cm) and was supplied with a somewhat different electronic system, having a wider band, with a readjustable central frequency. In addition, instruments were placed on the detector platforms for checking the force effects of nongravitational nature; seismographs, magnetometers, /107 acoustic pickups, and tiltmeters were used. Figure 24 shows a block diagram of the experimental outfit. The voltages from the piezoelectric transducers are fed to threshold detectors which are triggered by signals exceeding a certain level set by the experimenter. The shaped pulses are fed to a coincidence circuit; the latter produces a signal if the pulses arriving from both channels coincide in time. The time resolution τ_{res} in the first experiments was low, about 30 sec, but this was later brought to about 0.2 sec, i.e., the pulses were detected by the circuit as coinciding pulses if their leading edges were displaced in time by not more than $\tau_{res} \simeq 0.2$ sec.

Measurements with the coincidence circuit were made for several months. Several cases were recorded of coincidences of pulses exceeding the threshold level (approximately one per month). The threshold level was so much higher than the mean noise level that the probability of random coincidences for some cases was negligible (less than 0.0001). It is very important that according to the author the cases of coincidences were not accompanied by correlated /108 bursts on the other control instruments. On the basis of the exceedingly small probability of random coincidences, Weber rejects purely statistical reasons and feels that some rare synchronous effect was registered by the detectors, which, generally speaking, can be caused by gravitational radiation (!).

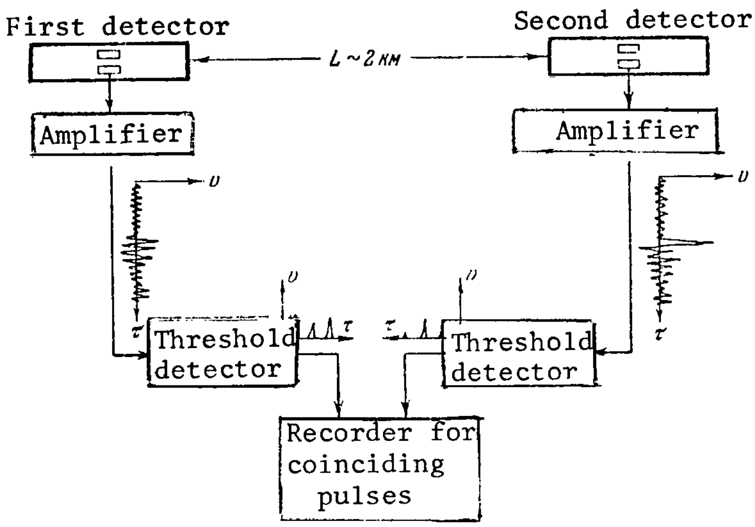

Figure 24

In order to evaluate the results better, we will once again return to the response of the Weber gravitational detector. We will examine a simplified model of a gravitational quadrupole in the form of two spaced masses connected by a spring. The relationship between R_{i0j0}, F^{μ}, t, etc. coincide in order of magnitude with the similar relationships for the case of an extended mass, differing only by insignificant factors.

The response of the Weber detector was limited by thermal fluctuations; these are described by the Nyquist theorem:

$$\overline{F^2_{fl}} \simeq 4\varkappa T H \Delta f,$$

where $\overline{F^2_{fl}}$ is the mean square fluctuation force, Δf is the frequency band for the receiver, H is the friction coefficient, related to the quality and masses entering the detector: $H = m\omega_0 Q^{-1}$. Substituting the fluctuation force into (8.13), in place of F_{gr} we obtain an expression for the minimum flux

t_{min} detectable under these conditions:

$$t_{\min} \simeq \frac{c^3}{2\pi G} \cdot \frac{\varkappa T}{Q m_{\text{equiv}} \omega_0 l^2_{\text{equiv}}} \Delta f. \tag{8.20}$$

Taking into account the parameters of this outfit: $\omega_0 \approx 10^4$ rad/sec, $Q \approx 10^5$, $T = 300°K$, equivalent mass $m_{equiv} \approx 5 \cdot 10^5$ g and length $l_{equiv} \approx 10^2$ cm, for t_{min} we obtain

$$t_{\min} \simeq 6 \cdot 10^5 \Delta f \frac{\text{erg}}{\text{sec} \cdot \text{cm}^2} \tag{8.21}$$

The receiver band in the described experiment was entirely determined by the coincidences circuit: different pulses shorter than 0.2 sec were not detected by the circuit, and therefore the equivalent band was about 5 cps. Accordingly, the absolute response was at the level $t \approx 3 \cdot 10^6$ erg/sec$\cdot$cm^2. Taking /109 into account that correlated bursts were observed when the threshold level was on the average approximately 10 times higher than the noise level, it must be assumed that the bursts corresponded to a flux $t \approx 3 \cdot 10^7$ erg/sec$\cdot$cm^2 (Weber gives $t \approx 2 \cdot 10^4$ erg/sec$\cdot$cm^2 [73], which corresponds to a threshold response of a detector with a band $\Delta\omega \approx 0.1$ rad/sec, which in this experimental system is not determining). A flux density $t \approx 3 \cdot 10^7$ erg/sec$\cdot$cm^2 is an extremely high value. Such a flux density for gravitational radiation at the earth's surface could be only from extremely exotic sources, such as a binary neutron star or an asymmetrically collapsing star [58, 61], situated at a distance not exceeding 1,000 light years from the earth. These estimates show that the response in the described gravitational detectors is substantially less than that which can be attained.

§ 9. Gravitational Relativistic Effects in the Nonwave Zone

The desirability of carrying out relativistic gravitational experiments in the nonwave zone is beyond question. However, with respect to obtaining new physical information these experiments are evidently less interesting than attempts to detect gravitational radiation of extraterrestrial origin, which, in addition to revealing the existence of this radiation, would provide a new source of astrophysical information.

Gravitational effects in the nonwave zone can be classified somewhat schematically into three groups: effects of interaction between electromagnetic radiation and the gravity field, nonlinear interaction of gravitational masses, and effects caused by the relative motion of gravitational masses. The execution of experiments pertaining to the first group of effects involves a method which is beyond the scope of this monograph. Further details concerning proposed and actually implemented experiments related to the first group of effects can be found in the review [77]. The detection of effects from the other two groups can be reduced to the detection of small forces or moments of forces acting on test bodies. Below we give estimates of the magnitudes of these effects and discuss the possibility of their observation from the point of view of limiting relationships in the optimum strategy of measurements described in Chapter II. /110

Nonlinear interactions of gravitational masses. A characteristic property of the Einstein equations is their nonlinearity. Even in the approximation of a weak field, the Lagrange function for n gravitating masses [54] contains a term describing the nonlinear interaction

$$\Delta L_{\text{nonlin}} \simeq \sum_a \sum_b{}' \sum_c{}' \frac{\gamma^2 m_a m_b m_c}{2c^2 |\mathbf{r}_{ab}||\mathbf{r}_{ac}|}, \tag{9.1}$$

where γ is the gravitational constant, c is the speed of light.

Using these expressions, it is easy to estimate the change in the attraction of the mass m_1 to the earth's mass M if there is still another mass m_2 at a distance r_{12} from m_1:

$$\Delta F_{\text{nonlin}} \simeq \frac{\gamma^2 m_1 m_2 M}{2c^2 r_1 R^2} = \frac{\gamma m_1 m_2 g}{2c^2 r_{12}}. \tag{9.2}$$

In formula (9.2) g is the acceleration due to free fall (in a linear approximation) at a distance R from the center of the earth. If it is assumed that $m_1 = m_2 = 10^4$ g, $r_{12} = 10$ cm, then $\Delta F_{nonlin} \simeq 3.5 \cdot 10^{-19}$ dyne. Using the limiting formula for the minimum detectable force (3.17) with the optimum strategy of measurements for $m_1 = 10^4$ g and assuming in (3.17) the oscillator strength in which m_1 enters to be equal to $\omega_{mech} = 10^{-3}$ sec^{-1}, we find that the necessary time $\hat{\tau}$ for detection of ΔF_{nonlin} is $\hat{\tau} \simeq 3 \cdot 10^5$ sec. We note that the ratio of the force ΔF_{nonlin} to the force of Newtonian attraction under these conditions is an extremely small value

$$\frac{\Delta F_{\text{nonlin}}}{F_{\text{New}}} \simeq \frac{g r_{12}}{2c^2} \simeq 1 \cdot 10^{-17}.$$

This means that when carrying out an experiment in which an attempt is made to detect ΔF_{nonlin}, the level of compensation for parasitic effects (relative to ΔF_{nonlin}) must be extremely high. If we compare this estimate /111 with that given in § 8 for an optimum detector, designed for the reception of radiation from close binaries, it can be concluded that in the case of small experimental scales it is approximately as difficult to detect the nonlinear interaction of three masses as it is to detect gravitational radiation.

Relativistic gravitational interaction of moving masses. The relativistic gravitational interaction of moving masses is similar to the interaction of moving electric charges (interaction of currents). This interaction is sometimes called prorotational. The addition ΔF_{prorot} to Newtonian attraction between two disks with the masses m_1 and m_2, caused by their rotation, is

$$\Delta F_{\text{prorot}} \simeq \frac{\gamma m_1 m_2 r^2 \omega^2}{c^2 r_{12}^2}. \tag{9.3}$$

The ratio

$$\frac{\Delta F_{\text{prorot}}}{F_{\text{New}}} \simeq \frac{r^2 \omega^2}{c^2} \simeq \frac{v^2}{c^2}$$

with $v = 3 \cdot 10^4$ cm/sec is equal in order of magnitude to $1 \cdot 10^{-12}$, i.e., is substantially greater than $\Delta F_{nonlin}/F_{New}$. Accordingly, it is considerably simpler to detect this effect than to detect the nonlinear interaction of gravitational masses. Schiff, Everitt and Fairbank are now carrying out an experiment [78, 79] in which the observation of the discussed effect is reduced to the observation of precession for a gyroscope installed on an artificial earth satellite. If the satellite orbit is 800 km from the earth's surface and in a polar orbit, as a result of satellite orbital motion its axis will be displaced by 7 seconds of angle each year, and due to the earth's rotation, by an additional 0.05 second of angle each year. The difficulties in executing this experiment are reduced essentially to creating a sufficiently stable indicator for the rotation of the gyroscope axis which would make it possible to implement remote measurements with an accuracy to about 0.01 second of angle each year (or 0.001 second of angle per month).

It is interesting to note that this effect (after its discovery and "mastery") must be taken into account in creating highly precise space navigation systems. /112

§ 10. Experiments With Test Bodies and Search for New Properties of Elementary Particles

Most detectors for individual nuclear reactions and detectors for high-energy elementary particles are, to use the expression employed by D. I. Blokhintsev in [30], "virtually unstable systems".

In § 4 we already pointed out that macroscopic oscillators with a small friction coefficient also can be employed in registering high-energy elementary particles. In this section we will briefly discuss an evaluation of the theoretically attainable response in two recently proposed macroscopic experiments: an experiment for the detection of the electric dipole moment of electrons and an experiment for detecting rare relict quarks with whole electric charges.

Macroscopic experiments for determining the dipole moment of an electron. The problem of the presence of electric dipole moments in elementary particles has recently acquired a timely nature due to the discovery of an apparent departure from T-invariance in some processes of the decay of neutral K-mesons (see review [80]).

It follows from various theoretical premises that the electric dipole moment of an electron d_e falls in the range $d_e \simeq 10^{-23}$-10^{-25} e·cm (where e is the electron charge). V. K. Ignatovich [81] (see also the review [80]) proposed a macroscopic experiment for detecting the electric dipole moment of atoms. The electric dipole moments of atoms in a nonrelativistic approximation (Schiff [82]) are equal to zero when the electrons have electric dipole moments. However, relativistic effects must lead to an intensification of the electric dipole moments (Sandars [83]). For example, for alkali atoms in the lower part of the Mendeleyev table of elements, the effective dipole moment of an atom d_{eff} must exceed by 2 orders of magnitude the dipole moment of an electron d_e. V. K. Ignatovich has proposed that d_{eff} be determined by magnetizing to saturation a nonconducting ferromagnetic substance with the number n of atoms in a unit volume. The atomic spins in this case will be completely oriented and if the electrons of these atoms have electric dipole moments there will be polarization of the electric sample $P = nd_{eff}$. This polarization corresponds to an electric field strength $E = 4\pi P\varepsilon^{-1}$, where ε is the dielectric constant of the medium. It is possible to determine d_{eff} by measuring the strength E of this field. When $d_{eff} = 10^{-21}$ e·cm, $n = 10^{22}$ cm^{-3}, $\varepsilon = 2$, the electric field strength arising due to such electric dipole moments will be $E_d \sim 10^{-5}$ V·cm. /113

Now we will estimate the theoretically attainable response in such an experiment from the point of view of the limitations set forth in Chapter II. We will assume that the sample of ferromagnetic material has the configuration of a sphere with the density ρ and the radius R. If the sphere is suspended on a fine filament in such a way that the magnetic field direction is perpendicular to the filament, and in addition to the magnetic field there is a superposed electric field E_0 perpendicular to the magnetic field and filament, the sphere will be acted upon by the moment of forces Mom F, which can swing such a torsional oscillator. The moment of forces can vary in rhythm with the oscillator oscillations, changing either the electric E or magnetic field. The amplitude of the moment of forces is

$$[\mathrm{Mom}\,F] \simeq \frac{4}{3}\pi R^3 n d_{\mathrm{eff}} E_0,$$

where E_0 is the amplitude of the superposed external electric field. If it is assumed that we can reduce the friction in the filament (in the suspension) to such a value at which the minimum torque is determined by the fluctuation effect in the optical indicator of small angular rotations, then

$$\frac{4\pi}{3} R^3 n d_{eff} E_0 \simeq \zeta \frac{2}{\hat{\tau}} \sqrt{\hbar \omega_{mech} I A'}. \quad (10.1)$$

Expression (10.1) follows from formula (3.25), in which the [Mom F] value has been substituted in place of the minimum detectable moment of forces. On the right-hand side of (10.1) ζ is a numerical factor of the order of several /114 units, determined by the selected level of detection reliability, $\hat{\tau}$ is the duration of the sinusoidal train during which the oscillator sways, $\hbar$ is the Planck constant, ω_{mech} is the frequency of torsional oscillations, I is the moment of inertia of the suspended sample, A' is a numerical factor characterizing the statistics of fluctuations in the optical source present in the indicator of torsional oscillations (for independently emitting photons $A' = 1$). Expression (10.1), like (3.25), is correct in the case of an optimum measurement strategy.

If we substitute into (10.1) $R = 1$ cm, $n = 10^{22}$ cm^{-3}, $E_0 = 10^2$ CGSE, $\zeta = 2$, $\hat{\tau} = 10^4$ sec, ω_{mech} 10^{-2} sec^{-1}, $A' = 10$ and $I = 8$ g$\cdot$cm^2 (which corresponds to $\rho \simeq 5$ g$\cdot$cm^{-3}), we obtain

$$[d_{eff}]_{min} \simeq 2.2\cdot 10^{-32} e\cdot \text{cm} \simeq 1.2\cdot 10^{-41} \text{ CGSE}. \quad (10.2)$$

If it is taken into account that the real attainable response in experiments with test bodies is approximately 7 orders of magnitude poorer, the value entirely attainable with present-day experimental equipment is $d_{eff} \simeq 2\cdot 10^{-25}$ e$\cdot$cm. This is approximately 2 orders of magnitude better than attained at the present time in other methods (see review [80]).

Macroscopic experiment for finding rare relict quarks with whole electric charges. Following formulation of the hypothesis by Gell-Mann and Zweig [43, 44] concerning the existence of quarks, whose electric charge should be $\pm$ 1/3 and $\pm$ 2/3 the charge of an electron, several competing hypotheses appeared in which it was postulated that quarks should have a whole (relative to the electron) electric charge.

According to one of these hypotheses (A. D. Sakharov [84]), relict quarks are accumulated at the center of massive stars and planets, experiencing only elastic scattering on the nuclei; their temperature corresponds to the temperature of the ambient medium (a thermalization of the quarks should occur). Their thermal distribution by density from the center of stars and planets to the periphery should lead to the following: near the surface there should be a nonzero quark concentration n_q. Near the earth's surface n_q can /115
fall in the range from 1 cm^{-3} to 10^6 cm^{-3}; the cross section of elastic scattering for such quarks on nuclei should be about 10^{-25} cm^2 and the mass should be about 13 proton masses [84].

A. D. Sakharov proposed a macroscopic experiment for finding such relict quarks under terrestrial laboratory conditions. The essence of this experiment was as follows: a massive torsional pendulum with axial symmetry on a torsion suspension is surrounded by a coaxially thick-walled cylinder which can be brought into rotation. The dimensions of the pendulum and the thickness of the cylinder are such that quarks incident on the pendulum, passing through the cylinder and pendulum, experience at least one collision. Thus, the cylinder rotation modulates the thermal velocities of quarks and imparts a torque to the pendulum. Swaying the cylinder first in one direction and then in the other, the pendulum can sway in rhythm. It is easy to estimate the amplitude of the torque [Mom F] which can be obtained in such an experiment:

$$[\mathrm{Mom}\, F] \simeq v_0 2\pi R^3 n_q \sqrt{13 \varkappa T m_p}. \qquad (10.3)$$

Here in (10.3) v_0 is the amplitude of the rate of cylinder rotation, R is the pendulum radius, equal to its height, κ is the Boltzmann constant, m_p is proton mass (the quark mass is $m_q \simeq 13\ m_p$). The R value must be sufficiently large in order that there will be an average of one quark collision with the pendulum nuclei. It is possible to avoid entrainment of the pendulum by the gas during cylinder rotation if they are separated by a fixed barrier which is quite thin and transparent for the thermal flux of quarks.

By equating (10.3) to the amplitude of the minimum detectable moment of forces (3.25) with an optimum measurement strategy, it is possible to estimate the threshold value n_q which in theory can be detected in such an experiment. Assuming in (10.3) that R = 30 cm, $v_0 = 3 \cdot 10^3$ cm/sec, T = 300°K and assuming in (3.25) that $\omega_{mech} = 10^{-2}$ sec^{-1}, $\hat{\tau} = 10^4$ sec, $\zeta = 2$, $A' = 10$, we obtain $n_q \simeq 2 \cdot 10^{-4}$ cm^{-3}, provided that the density of pendulum material is assumed to be $\rho \simeq 5$ g/cm^3. Since the real response in experiments with test bodies is approximately 7 orders of magnitude poorer, under present-day laboratory conditions it would be possible to expect a response in such an /116
experiment with a corresponding concentration $n_q \simeq 10^3$ cm^{-3}.

As in the case with the electric dipole moment of electrons, we will not discuss the detailed requirements for equipment in such experiments and will not examine the necessary control experiments, since this is beyond the scope of this book.

Summarizing the material examined in Chapter IV, it can be concluded that experiments with test bodies unquestionably have substantial advantages in a great number of investigations. The author has not attempted to cover all possible fields of applicability of experiments in which the detection of a physical effect is essentially reduced to the detection of a small force or moment of forces. The examples given in Chapter IV should instead be regarded as illustrations and evaluations of the attainable response in those experiments in which fundamental physical problems can be solved.

§ 11. Methods for Measuring Small Mechanical Oscillations

Radio engineering methods. Radio engineering methods, making it possible to measure small mechanical displacements and mechanical oscillations, are very readily employable under ordinary laboratory conditions and make it possible to obtain a high response. When measuring small quasistatic displacements with capacitive transducers it is possible to register displacements of $\Delta x \simeq 10^{-9}$ cm [85]; measurement of small mechanical oscillations of sonic frequencies, also by capacitive transducers, makes possible the reliable discrimination of an amplitude of oscillations $x_0 \simeq 6 \cdot 10^{-13}$ cm for a time of signal discrimination from noise of about 200 sec. It is clear that the limiting response in measuring quasistatic displacements is determined for the most part by the extent to which it is possible during the experiment to thermostabilize the instrumentation, and especially the mechanical objects whose relative displacement is under investigation. Measurement of variable mechanical displacements is more interesting from the point of view of applying these methods in experiments with test bodies.

The most sensitive among the various radio engineering devices for transforming mechanical movements into electric signals are so-called capacitive transducers. The plates of an electric capacitor, the change in distance d between which should be measured, together with an inductance, form an electric circuit. This circuit either is included in a radio-frequency oscillator, or electric oscillations are excited in the circuit by a supplementary self-excited oscillator. With a change in d, in the first case there /118
is a change in the frequency of the generator; in the second case there is a change in the amplitude of the oscillations in the circuit (in this case the frequency of the self-excited oscillator is usually tuned on the "slope" of the resonance curve). Simple radio engineering devices register these changes.

The possibilities of a method are limited primarily by the frequency and amplitude fluctuations of the self-excited oscillator. The amplitude fluctuations can be considerably attenuated (for example, see [86]). In registering the changes in distance between the plates $x(\tau)$ it is necessary that the change in the characteristic frequency of the circuit δf caused by this displacement be greater than the fluctuation drift of the generator frequency[1]:

$$\delta f = \alpha f_0 \frac{x(\tau)}{d} \geqslant \zeta \sqrt{W(f)\Delta f}. \tag{11.1}$$

In this expression $\alpha \lesssim 0.5$ (for real circuits, having a stray capacitance $\alpha \simeq 0.3$-0.4), $W(f)$ is the spectral density of frequency deviations for the self-excited oscillator, Δf is the frequency band characteristic for $x(\tau)$, and ζ is a value of about several units, determined by the selected level of detection reliability. If it is assumed that the width of the self-excited oscillator line determines only the shot effect, by using known expressions for $W(f)$ (for example, see [87]), we can obtain an estimate for the minimum detectable $x(\tau)$ value:

$$x(\tau) = \zeta d\alpha^{-1} [eI_0 r\Delta f(2N)^{-1}]^{1/2}, \tag{11.2}$$

where I_0 is the constant component of the plate current in the tube, e is the electron charge, r is the resistance in the oscillator circuit, N is the oscillator power.

Now we will estimate the minimum distinguishable displacement $x(\tau)$ for some specific values of the parameters entering into (11.2). Assume $\zeta = 2$ (for the reliability level about 0.95), $r \simeq 10^{-3}$ ohm (for "superconductive" alloys at the temperature of liquid helium $T = 4°K$ and at a frequency $f_0 = 10^5$ cps [88]), $d = 10^{-2}$ cm, $I_0 = 10^{-4}$ a, $N = 10^{-4}$ W, $e = 1.6 \cdot 10^{-19}$ Coulomb. Then /119

$$[x(\tau)]_{0.95} \simeq 3.6 \cdot 10^{-13} \sqrt{\Delta f}\ \text{cm}$$

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[1] Here we assume that the Δf band is sufficiently small so that $W(f)$ does not change significantly within Δf.

The cited estimate for $[x(\tau)]_{0.95}$ should evidently not be regarded as the limiting value. If it is possible to use one oscillator for exciting oscillations in two circuits, and the distance between the plates changes in only one of them, by using the ordinary compensation method it is possible to reduce the $[x(\tau)]_{0.95}$ value by a factor 10--10^{-4} (this is the usual figure for circuits with compensation). In this case it will be both frequency and amplitude fluctuations which are compensated. In this type of instrumentation the level of detectable $x(\tau)$ can be affected by incoherent thermal oscillations in the circuit. At the end of this section we will return to the problem of the possibility of a substantial narrowing of the natural line width of the source of radio-frequency oscillations and the role of incoherent thermal fluctuations in circuits.

We note that in the discussed method a substantial role is played by instrument noise; this noise determines the lower limit of measured fluctuations in frequency deviation and accordingly, small mechanical displacements. However, the level of instrument noise is also dependent on the "refinement" of the experiment (on the extent to which it has been possible to reduce the flicker effect, microphone noise, noise level of mixers, etc.), whereas fluctuations in oscillator frequency caused by the shot effect are essentially unexcludable. The effect of amplitude fluctuations is less than the influence of frequency fluctuations; this is easily confirmed by using /20
known expressions for the amplitude fluctuations of a self-excited oscillator.

Figure 25 shows the results of measurements of small amplitudes of oscillations of a tuning fork at a frequency of 15 cps produced by means of a capacitive transducer [89]. The transducer had a compensation circuit with a compensation factor $\beta = 1 \cdot 10^{-2}$. Each point on the graph corresponds to a discrimination time of about 20 sec. The amplitudes of the oscillations were plotted along the x-axis, whereas the galvanometer readings at the transducer output were plotted along the y-axis. The amplitudes of the oscillations were determined (calibrated) using the known mechanical parameters of the tuning fork and the known force applied to the tuning fork (for further details see [89]).

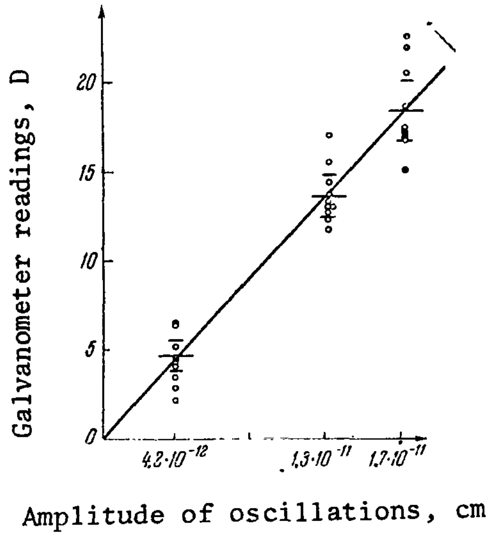

Figure 25

Now we will briefly discuss the possibilities of increasing the resolution of this method. We note that the real time during which a self-excited oscillator is necessary for the capacitive transducer does not exceed 10^6--10^5 sec under ordinary physical conditions. This means that a standard oscillator theoretically could be interchangeable with a linear electric oscillatory system with a time constant $\tau^* \gtrsim 10^7$ sec.

With such a source of oscillations the line width would be determined only by thermal fluctuations. However, in the best present-day quartz resonators in the radio-frequency range the attenuation constant is several seconds [90]. A massive rotating rotor, suspended in a vacuum by means of a so-called magnetic suspension, can be used as a linear system with large τ^*. Under such conditions the rotation of the rotor is slowed due to slight friction caused by rarified gas surrounding the rotor. For example, in the Beams ultracentrifuge [91], suspended in a magnetic field in a vacuum (T = 300 °K, $p = 10^{-6}$ mm Hg), the steel rotor (m = 13 kg) was slowed, losing about 10 rpm/day (i.e., during $\tau^* \simeq 10^5$ sec) in rotation frequency, provided it was imparted a rotation frequency of about 2×10^4 rpm. Such a rotor, together with a rotation frequency detector (for example, optical or capacitive), can be used as a source of radio-frequency or sonic signals. The maximum frequency of rotor rotation is determined by the ultimate strength of the material from which the rotor is fabricated and /121
can attain $f_0 \simeq 10^6$ cps [91]. The slight frequency drift of such an apparatus is easily compensated by small energy pumping (for example, by means of light pressure); using well-known automatic frequency control methods it is possible to stabilize the mean frequency of rotor rotation using a stable self-exciting oscillator.

It is easy to compute the spectral density of rotor rotation frequency deviation caused by fluctuations of gas molecule pressure on the rotor. Assuming that the rotor has the form of a cylinder with an altitude and radius equal to a and a mass m, for the spectral density of rotation frequency deviation (without allowance for frequency drift) we obtain

$$W_{\mathrm{rot}}(\Omega) \simeq \frac{D(\Omega)}{I^2\Omega^2 + H_\omega^2} = \frac{32\mu^{1/2}(\varkappa T)^{3/2} n}{\pi\sqrt{2\pi}\,[(9/4)\,m^2\Omega^2 + 128\pi\mu\varkappa T n^2 a^4]}\,. \tag{11.3}$$

Here $D(\Omega)$ is the spectral density of fluctuations of the pressure moment on the rotor, I is the rotor moment of inertia, H_ω is the rotor friction coefficient for rotational movement, μ is the mass of a gas molecule, n is the concentration of gas molecules, κ is the Boltzmann constant; Ω is reckoned in rad/sec from the rotor frequency of rotation $2\pi f_0$. Expression (11.3) was obtained under the following assumptions: a) $2\pi a f_0 \ll (\kappa T/\mu)^{1/2}$, where f_0 is the mean frequency of rotation; b) Ω is less than the lowest characteristic frequency of rotor mechanical oscillations; c) $\Omega\hat{\tau} \gg 1$. Since $\hat{\tau} \ll \tau^* = I/H_\omega$ and we are interested in such frequencies for which $\Omega\hat{\tau} \gg 1$, in the considered example we see a random process with a stationary increment [92]. Accordingly, expression (11.3), strictly speaking, is a Fourier transform of the structural function, which, as is well known, coincides with the usual spectral density of a stationary process, provided $\Omega\hat{\tau} \gg 1$.

The expression for $W_{rot}(\Omega)$ differs in structure from the expression for the spectral density of frequency deviation for a tube generator. The greater the Ω and m values, the smaller is the T value and the better is the vacuum (i.e., the lesser the n value), the lesser is the $W_{rot}(\Omega)$ value. Thus, the level of experimental equipment will determine the line width of this signal source. If in (11.3) we substitute $\mu = 2.7 \cdot 10^{-23}$g, $T = 300$°K, /122
$a = 3$ cm, $n = 2.7 \cdot 10^{10}$ cm^{-3} (i.e., with $p = 10^{-6}$ mm Hg), $\Omega = 60$ rad/sec, $m = 6 \cdot 10^2$ g. then $W_{rot}(\Omega) = 1.5 \cdot 10^{-28}$ rad/sec. For a tube generator with $f_0 = 10^4$ cps (i.e., approximately with the same frequency as for the rotor in the preceding estimate), $I_0 = 10^{-3}$ a, $r = 1$ ohm and $N_0 = 10^{-2}$ W, we obtain $W_{t.o.}(\Omega) = 1.3 \cdot 10^{-10}$ rad/sec (where the subscript $t.o.$ refers to the tube oscillator). Thus, at least under the described conditions the difference

between W_{rot} (Ω) and $W_{t.o.}$ (Ω) is 18 orders of magnitude. In this estimate for W_{rot} (Ω) smaller n and T values were not used because the W_{rot} (Ω) can increase the fluctuations caused by viscous magnetic friction in the suspension. Simple computations on the basis of data in [91] reveal that at least when $p = 10^{-6}$ mm Hg and $T = 300°$K the magnetic suspension does not introduce any substantial additional attenuation into the torsional motion in comparison with the friction caused by the residual gas pressure. Accordingly, on the basis of the generalized Nyquist theorem it can be concluded that the estimate given above for W_{rot} (Ω) is correct. There is still another mechanism for the increase in the fluctuation frequency deviation of rotor rotation caused by its own characteristic mechanical thermal oscillations. Simple computations, which we will not present here, show that the contribution of thermal oscillations to the W_{rot} (Ω) value, estimated using expression (11.3), is insignificant.

In summarizing these considerations, it can evidently be asserted that a considerable attenuation of the effect of source frequency fluctuations on resolution can be achieved either by replacing the radio-frequency self-excited oscillator by a device similar to that described above, or by increasing the degree of compensation. If it is assumed that by the use of such methods it is possible to eliminate completely the effect of source frequency fluctuations on resolution, the latter can be determined only by the presence of incoherent thermal electric oscillations in the circuits. The minimum oscillations which can be resolved in this case, are determined using the simple expression

$$[x(\tau)]_{\min} = \zeta \frac{d}{\alpha U_{\sim}} \sqrt{4\varkappa T r \Delta f}, \tag{11.4}$$

where, as in (11.2), $\alpha \lesssim 0.5$, ζ is a dimensionless factor of the order of several units, r is the resistance of the circuit in which the sensor capacitance is included, κ is the Boltzmann constant, T is the circuit temperature, $U_{\sim}$ is the amplitude of the electric voltage in the circuit, Δf is the frequency band characteristic for $x(\tau)$. Assuming $U_{\sim}/d = 10^5$ V/cm (which is admissible if there is a sufficiently high vacuum between the /123 sensor capacitor plates), $T = 4°K$, $r = 10^{-3}$ ohm, $\zeta = 2$, $\alpha = 0.5$, we obtain $[x(\tau)]_{0.95} \simeq 1.8 \cdot 10^{-17} \sqrt{\Delta f}$ cm. We note that if it is possible to make long-term measurements and $\Delta f \ll f_{mean}$, in expressions (11.3) and (11.4) it is necessary to replace Δf by $\sqrt{\Delta f/\hat{\tau}}$, where $\hat{\tau}$ is the time expended in signal discrimination. There is also some change in the dimensionless factor ζ (for example, see [101]). The latter estimate for $[x(\tau)]_{0.95}$ must evidently be regarded as limiting for radio engineering methods for measuring small mechanical oscillations.

Optical methods. In various modifications there are two principal optical methods for detecting small mechanical displacements or oscillations. In the first method (it is sometimes called the "knife and slit" method or optical lever method [13, 14]) the optical image of one diffraction grating, obtained using an objective, is matched with a second grating having the same interval. The displacement of one of these gratings parallel to the other causes a modulation of the light flux passing through the two gratings. This modulation can be registered by a photodetector. The lesser the grating interval, the more intense is the modulation for the same displacement. If it is assumed that the grating interval is of the order of the wavelength, the light flux fluctuations are caused only by the independence of emission of photons from the source, and the quantum yield of the photodetector is close to unity, the minimum displacement $[x(\tau)]_{min}$ which can be registered by such a device is determined from the condition

$$\frac{[x(\tau)]_{min}}{\lambda_0} \gtrsim \sqrt{\frac{2h\nu_0}{N_0}\Delta f},$$

or /124

$$[x(\tau)]_{min} \simeq \zeta\lambda_0 \sqrt{\frac{2h\nu_0}{N_0}\Delta f} = \zeta \sqrt{\frac{2hc\lambda_0}{N_0}\Delta f}, \quad (11.5)$$

where h is the Planck constant, N_0 is the intensity of the light flux after the second diffration grating, Δf is the frequency band characteristic for $x(\tau)$, λ_0 and ν_0 are the wave length and the emission frequency respectively, ζ is a dimensionless factor of the order of several units, dependent on the selected level of detection reliability. If it is assumed that $\zeta = 2$, $\lambda_0 = 5 \cdot 10^{-5}$ cm, $N_0 = 10^6$ erg/sec, then $x(\tau) \approx 1.7 \cdot 10^{-13} \sqrt{\Delta f}$ cm. The minimum quasistatic displacement which Jones could register by this method was several units times 10^{-12} cm [14]. The entire mechanical part of the optical system was thermostabilized to about $1 \cdot 10^{-6}$ °C.

In place of two diffraction gratings, in order to obtain an intense modulation of the light flux it is possible to use an optical knife which covers the light flux near the focal spot of the optical objective. If aberrations are eliminated in the objective, and the light source gives a plane monochromatic wave, the distribution of intensity of optical radiation near the objective focus is determined only by wave diffraction in the objective aperture. Using known expressions for the light wave field near a focal spot [93], it is easy to compute the minimum displacement $x(\tau)$ of the optical knife which can be detected:

$$x(\tau) \simeq 0.2\zeta \frac{L}{a} \lambda_0 \sqrt{\frac{2h\nu_0}{N_0} \Delta f} = 0.2\zeta \frac{L}{a} \sqrt{\frac{2hc\lambda_0}{N_0} \Delta f}, \qquad (11.6)$$

where L is the objective focal length, a is the diameter of the objective entrance aperture. As can be seen from a comparison of (11.5) and (11.6), the basic characteristic determining the minimum detectable displacements is the spectral density of fluctuation modulation of the light flux intensity M_f^2. In deriving expressions (11.5) and (11.6) it was assumed that the emission of individual photons from the source occurs independently, and therefore /125

$$M_f^2 = \frac{2h\nu_0}{N_0}. \qquad (11.7)$$

Evidently, until now it has not been possible to create a source of optical radiation with a M_f^2 value less than (11.7). However, there are no theoretical limitations on decreasing the M_f^2 value with this same flux intensity N_0. Using nonlinear optical systems[1], it is evidently possible to obtain a

[1] For example, systems similar to parametric dampers in the radio-frequency range [86].

substantial decrease in the M_f^2 value, and accordingly decrease the minimum detectable displacements by means of this method. We will return to the problem of the possibilities of decreasing M_f^2 later in this chapter.

The second optical method for measuring small mechanical displacements involves the registry, by means of a photodetector, of changes in the light flux passing through an interferometer during the movement of its mirrors. Using the Michelson interferometer, employing a mercury tube as the source of one of the spectral lines, I. L. Bershteyn [94] succeeded in registering oscillations of mirrors at a sonic frequency with an amplitude of about 10^{-11} cm. Javan [15] feels that by attaining frequency stability of helium-neon gas lasers it will be possible to use a Fabry-Perot resonator to register relative displacements of its mirrors of the order of several units per 10^{-13} cm.

We will examine the limiting sensitivity of this method using the example of a Fabry-Perot resonator. We will assume that the emission from a laser with the power N_0 and the frequency ν_0, operating in a single-mode regime, excites optical oscillations in the Fabry-Perot resonator in the fundamental mode. A photodetector with a quantum yield close to unity registers the modulation of the laser light flux passing through the resonator. Such resonators, in whose mirrors multilayer dielectric coatings are used, have an extremely high quality /126

$$Q_{opt} = \frac{\nu_{res}}{\Delta\nu_{res}} = \frac{2\pi\nu_{res}l}{c(1-R)}, \tag{11.8}$$

where l is the distance between the mirrors, c is the speed of light, R is the coefficient of reflection from the mirror. Present-day multilayer coatings make it possible to obtain $R \simeq 0.995$, and accordingly the Q_{opt} value attains 10^{10}. We will assume that the resonator is detuned relative to the laser frequency by the value $\nu_{res}/2Q_{opt}$. In this case one attains a nearly maximum intensity modulation for a light flux passing through the resonator with a small relative displacement of the mirrors. If we use known expressions for the shape of the resonance curve for such a resonator (for example, see [95]), it is possible to compute the relative change in light flux intensity at the resonator output $M_{x(\tau)}$ caused by the displacement $x(\tau)$ of one mirror relative to the other[1]:

$$M_{x(\tau)} = \frac{2\pi x(\tau)}{(1-R)\lambda_0}. \qquad (11.9)$$

We will assume that the laser frequency does not fluctuate, but the fluctuations of laser intensity are the same as for a light source from which the photon yield is independent. Then the detection condition $x(\tau)$ has the simple form:

$$M_{x(\tau)} = \zeta \sqrt{M_f^2 \Delta f}.$$

Hence, using (11.7), we obtain

$$[x(\tau)]_{\min} = \zeta \frac{(1-R)\lambda_0}{2\pi} \sqrt{\frac{2h\nu_0}{N_0}\Delta f}, \qquad (11.10)$$

where, as before, ζ denotes a dimensionless factor of the order of several units, dependent on the selected level of detection reliability. As can be seen from a comparison of (11.5), (11.6) with (11.10), the increase in resolution for the second method in comparison with the first is approximately /127
$(1 - R)^{-1}$ times.

Now we will take into account fluctuations in laser frequency which also lead to a fluctuation modulation of the light flux amplitude at the

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[1] Expression (11.9) for $M_{x(\tau)}$, derived on the assumption that the resonator is detuned relative to the laser frequency by $\nu_{res}/2Q_{opt}$, gives a value approximately 20% less than $[M_{x(\tau)}]_{max}$, which is obtained with a somewhat different detuning.

resonator output. For this purpose we use an approximate expression for the spectral density of laser frequency deviation [95][1]:

$$W(f) \cong \frac{8\pi h\nu_0 (\Delta\nu_{\text{resonat}})^2}{N_0}. \tag{11.11}$$

Modulation of the characteristic frequency of the Fabry-Perot resonator, caused by displacement of the mirrors $\Delta\nu_{mod}/\nu_{resonat} = x(\tau)/l$, should be greater than $\sqrt{W(f)\Delta f}/\nu_0$. Therefore

$$[x(\tau)]_{\min} = l\frac{\Delta\nu_{\text{mod}}}{\nu_{\text{resonat}}} = \zeta l \frac{\sqrt{W(f)\Delta f}}{\nu_0}. \tag{11.12}$$

Using (11.11), as well as the condition $\nu_{resonat} \simeq \nu_0$, we obtain

$$[x(\tau)]_{\min} = \zeta(1 - R)\lambda_0 \sqrt{\frac{2h\nu_0}{\pi N_0}\Delta f}. \tag{11.13}$$

Expressions (11.13) and (11.10) differ only in the factor $\sqrt{2\pi}$. Assuming $\zeta = 2$, $R = 0.995$, $N_0 = 10^6$ erg/sec, $\lambda_0 = 5\cdot 10^{-5}$ cm, we obtain $[x(\tau)]_{0.95} \simeq 7\cdot 10^{-16}\sqrt{\Delta f}$ cm.

In the derivation of (11.13) and (11.10) approximate expressions were used for $W(f)$ and M_f^2 for a source with fluctuations independent of the emitting photons.

I. L. Bershteyn, I. A. Andronova and Yu. A. Zaytsev demonstrated in [25] that in deriving an expression for M_f^2 for a laser it is necessary to take into account the rigidity of the limiting cycle, as well as the nonlinear and dispersion properties of the active medium in the laser. Taking into account the rigidity of the limiting cycle, the expression for M_f^2 for a laser has the /128

[1] Formula (11.11) is an approximate expression for the natural line width $\Delta\nu_{nat}$ of an optical self-excited oscillator. V. S. Troitskiy demonstrated in [96] that the natural line width of a self-excited oscillator $\Delta\nu_{nat} = W(0)$. Accordingly, (11.11) is correct only for low frequencies.

following form [25]:

$$M_f^2(f) \simeq \frac{2h\nu_0}{N_0} \frac{(\Delta\nu_{resonat})^2}{(p^2+f)}, \tag{11.14}$$

where p^2 is the rigidity of the limiting cycle, f is the frequency, reckoned from ν_0. In modern optical oscillators $\Delta\nu_{resonat} \approx 10^6$ cps, and $p = 10^5$ cps. Accordingly, the experimentally observed M_f^2 values at low frequencies are approximately two orders of magnitude greater than for a source with independent fluctuations [24]. This means that the minimum detectable mechanical displacements will be somewhat greater than the estimates which can be obtained using (11.13) and (11.10).

It can be seen from (11.14) that if it is possible to create a resonator with $\Delta\nu_{resonat} < p$, it will be possible to have a source in which the intensity fluctuations are less than in a source with an independent emission of photons. A similar theoretical possibility also exists with respect to the $W(f)$ value (for further details see [25]).

As can be seen from the above, optical methods for measuring small mechanical displacements have a high resolution which is essentially dependent on the properties of the emission sources. It evidently is impossible to assert, as in the case of radio engineering methods for measuring small displacements, that in the already performed experiments a real limit of resolution of these methods has been attained.

§ 12. Mechanical Fluctuations in a Space Laboratory

On the basis of the available, quite extensive experimental data, obtained using already launched space stations, we can obtain approximate estimates which make it possible to judge to what degree it is possible to approach the theoretically attainable response in an optimum measurement strategy (see Chapter I) when formulating experiments with test bodies. The vacuum in a space laboratory, assuming an adequate distance from the earth, is much better than that which is usually attained in a terrestrial labora- /129
tory. The friction coefficient H_{gas} corresponding to this vacuum, even for small test bodies, is quite small, and when carrying out an experiment with

a test body under such conditions it is already possible to require the use of a small displacements detector (optical or electronic) which has optimum tuning. Accordingly, in this section we will be concerned only with the fluctuations of the center of mass of a space laboratory, equivalent to the seismic oscillations of a terrestrial laboratory. Such fluctuations can be caused by fluctuations in pressure of the solar wind, fluctuations in the magnetic field, micrometeorites, or movements of another space laboratory. It is convenient to compare the accelerations of the center of mass of a space laboratory caused by different fluctuation factors and the theoretically measurable periodic acceleration under optimum strategy conditions. This comparison will be given below for specific parameters in a hypothetical experiment with a test body.

Fluctuations of solar wind pressure. At the present time complete information is unavailable concerning the spectrum of solar wind fluctuations. Rather detailed information is available concerning the diurnal and hourly variations in solar wind intensity (see review [97]), but for higher frequencies of variations no measurements have yet been made. Below we will give three estimates, making it possible to compare the accelerations imparted by the solar wind to a space station and the periodic accelerations of a mechanical oscillator detectable when optimum strategy is employed.

The solar wind pressure θ at the distance of one astronomical unit from the sun (in "quiet weather") is about $4 \cdot 10^{-9}$ dyne/cm^2 [98]. This means that a space station with the mass $M = 10^7$ g and the cross section $S = 10^5$ cm^2 is accelerated by the solar wind with a mean acceleration $\ddot{r} = \theta S/M \simeq 4 \cdot 10^{-11}$ cm/sec^{-2}.

We will assume that aboard the space station an experiment is carried out for detecting the periodic acceleration of the mass m of a mechanical oscillator having the frequency ω_0. It follows from formula (3.17) that the minimum detectable amplitude of acceleration with the frequency ω_0 during the time $\hat{\tau}$ is /130

$$\left[\frac{F}{m}\right]_{\min} \simeq \frac{2}{\tau} \sqrt{\frac{\hbar\omega_0 A}{m}}. \qquad (12.1)$$

If $A = 10$, $\hat{\tau} = 10^3$ sec, $\omega_0 = 1$ sec^{-1}, $m = 10^2$ g, then $[F/m]_{min} \simeq 2 \cdot 10^{-17}$ cm·sec^{-2}. This value must be compared with the mean square fluctuation acceleration $\sqrt{(\ddot{r})^2_\omega \Delta f}$, caused by solar pressure in the frequency band $\Delta f \simeq 1/\hat{\tau}$:

$$\sqrt{(\ddot{r})^2_\omega \Delta f} = \sqrt{\frac{2\dot{\theta} S m_p v \Delta f}{M^2}}, \tag{12.2}$$

where v is the velocity of solar wind particles, m_p is proton mass.

Expression (12.2) is correct if the impacts of protons against the outside of the space station are considered uncorrelated. Assuming in (12.2) that $v = 3 \cdot 10^7$ cm·sec^{-1} and $\Delta f = 10^{-3}$ sec^{-1}, we obtain $\sqrt{(\ddot{r})^2_\omega \Delta f} \simeq 6 \cdot 10^{-19}$ cm·sec^{-2}, i.e., one and one-half orders of magnitude less than $[F/m]_{min}$ in the estimate given above. With an increase in $\hat{\tau}$ the estimates obtained from (12.1) and (12.2) are comparable since in (12.1) $[F/m]_{min}$ decreases as $\hat{\tau}^{-1}$, whereas in (12.2) $\sqrt{(\ddot{r})^2_\omega \Delta f}$ decreases as $(\hat{\tau})^{-1/2}$.

Thus, the hypothetical conclusion can be drawn that in the case of not excessively great durations $\hat{\tau}$ it is possible to attain the maximum response in experiments with test bodies without using any special screens for decreasing the fluctuations of solar wind pressure on a space station.

Magnetic field in circumsolar space. The measurements made on heliocentric stations revealed that far from the earth, at a distance of about one astronomical unit from the sun, the magnetic field is 10^{-4}-10^{-5} oe and its variations with approximately the same amplitude have characteristic periods of about 1 hour (i.e., grad $B \simeq 10^{-14}$-10^{-15} oe/cm) [98]. A nonmagnetic space station in a uniform magnetic field has the acceleration /131

$$\left[\frac{F}{m}\right]_{\text{mag}} \simeq \frac{\chi}{\rho} B \frac{\partial B}{\partial r}, \tag{12.3}$$

where χ is the mean permeability, ρ is the mean station density. The acceleration is $[F/m]_{mag} \simeq 3 \cdot 10^{-24}$ cm·sec^{-2}, provided that $\chi \simeq 10^{-5}$, $\rho = 3$ g·cm^{-3}. As can be seen from a comparison of this estimate $[F/m]_{mag}$ and the above-cited value $[F/m]_{min} \simeq 2 \cdot 10^{-17}$ cm·sec^{-2}, in some hypothetical experiment with a test body, in the case of a nonmagnetic station the possible accelerations caused by magnetic field fluctuations in space can be neglected. However, the existence of ferromagnetic parts in the station can greatly increase $[F/m]_{mag}$.

Fluctuations in the acceleration of a space station caused by micrometeorites. In order to use an expression similar to (12.2) in computing the mean square acceleration $\sqrt{(\ddot{r})^2_{\omega}\Delta f}$ of the center of mass of a space station caused by multiple impacts of micrometeorites, it is necessary to take into account from the entire spectrum of micrometeorites only those which during the time $\hat{\tau}$ impact on the station a sufficiently great number of times. Micrometeorites with the mass $\hat{m} \simeq 10^{-11}$ g collide with a station with the area $S = 10$ m^2 on an average of five times during $\hat{\tau} = 10^3$ sec; if the mass is $\hat{m} \simeq 10^{-12}$ g, the average is 50 times, whereas if the mass is $\hat{m} \simeq 10^{-10}$ g the possibility of such a collision is about 0.5.

The expression for shot fluctuations in the acceleration of a space station center of mass, similar to (12.2), has the form

$$\sqrt{(\ddot{r})^2_{\omega}\Delta f} \simeq \sqrt{\frac{2v\Delta f \sum\limits_i \bar{F}_i \hat{m}_i}{M^2}}. \qquad (12.4)$$

where M is station mass, $\bar{F}_i = \hat{m}_i \Phi_i S$ is the mean pressure exerted on one side of the station by micrometeorites having the mass $\hat{m}_i$ and the collision cross section Φ_i. In (12.4) summation by meteorite masses must be limited by the condition $S\Phi_i\tau \gg 1$. More infrequent impacts of more massive meteorites naturally also make a contribution to the fluctuation acceleration of the space station center of mass. However, rare strong acceleration bursts can be registered separately, using for this purpose instruments designed for registering micrometeorites, and then their influence on the motion of the test body can be taken into account. The contribution to acceleration by frequent impacts of small meteorites is evidently considerably more difficult to take into account. If we employ the summarized data on Φ_i and $\hat{m}_i$ given in [99, 100], then for $\hat{\tau}_i = 10^3$ sec and $S = 10$ m^2 we find that the maximum $\hat{m}_i$ at which it is possible to use (12.4) is about $1 \cdot 10^{-11}$ g. Assuming in /132

(12.4) that $M = 10^7$ g, $v = 30$ km/sec, and using the data on Φ_i and $\hat{m}_i$ from [100], with $\Delta f = 10^{-3}$ sec^{-1} we obtain a value $\sqrt{(\ddot{r})^2_\omega \Delta f} \simeq 4 \cdot 10^{-15}$ cm/sec^2. This is more than two orders of magnitude greater than the value $[F/m]_{min} \simeq 2 \cdot 10^{-17}$ cm/sec^2 obtained under the conditions described above. Thus, the fluctuations in micrometeorite pressure make it difficult to attain the maximum sensitivity if experiments are carried out with test bodies on a space station.

A possible solution of this problem involves the use of an anti-meteorite screen surrounding the station. In this case an effect of micrometeorites on the test body will be exerted only through a variable gravity field which appears as a result of oscillations of the screen from meteorite impacts. Such a screen can be designed similar to antiseismic filters which are employed in terrestrial laboratories.

In conclusion, we will cite still another numerical estimate characterizing the conditions necessary for attaining a response corresponding to the above-mentioned estimate for $[F/m]_{min}$. If the information on acceleration of a test body in one space station is transmitted to another space station, it is necessary to take into account the acceleration experienced by the first station in the gravity field of the second. We will assume that the second station also has the mass $M \simeq 10^7$ g and the test body m_0 is displaced relative to the center of mass of the first station by 1 m. Then the constant acceleration experienced by the test body relative to the center of mass of the first space station (this acceleration also will be registered by a small oscillations detector) will be about $1 \cdot 10^{-17}$ cm$\cdot$sec^{-2}, provided the stations are 10 km apart.

REFERENCES

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1. Chandrasekar, S., *Stokhasticheskiye problemy v fizike i astronomii* [Stochastic Problems in Physics and Astronomy], "Inostrannaya Literatura" Press, 1947.
2. Granovskiy, V. L., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 13, No. 805, 1933.
3. Leontovich, M. A., *Statisticheskaya Fizika* [Statistical Physics], "Gostekhizdat" Press, 1944.
4. Bunimovich, V. I., *Fluktuatsionnyye Protsessy v Radiopriyemnykh Ustroystvakh* [Fluctuation Processes in Radio Receivers], "Gostekhizdat" Press, 1950.
5. Stratonovich, R. L., *Izbrannyye Voprosy Teorii Fluktuatsiy v Radiotekhnike* [Selected Problems in the Theory of Fluctuations in Electronics], "Sovetskoye Radio" Press, 1961.
6. Khal'd, A., *Matematicheskaya Statistika s Tekhnicheskimi Prilozheniyami* [Mathematical Statistics with Technical Applications], "Inostrannaya Literatura" Press, 1956.
7. Braginskiy, V. B. and V. K. Martynov, *VMU* [*Vestnik Moskovskogo Universiteta*], *Ser. III*, No. 2, p. 60, 1966.
8. Braginskiy, V. B., *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 53, p. 1436, 1967.
9. Braginskiy, V. B., I. I. Minakova and P. M. Stepunin, *PTE*, No. 3, p. 183, 1965.
10. Braginskiy, V. B., *VMU* [*Vestnik Moskovskuco Universiteta*], *Seriya III*, No. 2, p. 65, 1965.
11. Braginskiy, V. B. and I. I. Minakova, *VMU* [*Vestnik Moskovskuco Universiteta*], *Seriya III*, No. 1, p. 69, 1964.
12. Karliner, M. M., V. E. Shapiro and I. A. Shekhtman, *ZhTF* [*Zhurnal Tekhnicheskoy Fiziki*], Vol. 36, p. 2017, 1966.
13. Vasil'yev, L. A. and O. M. Sineglazov, *Optika i Spektroskopiya*, Vol. 18, p. 1065, 1965.
14. Jones, R. V. and J. C. Richards, *J. Sci. Instr.*, Vol. 36, p. 90, 1959.
15. Javan, A., Lasers and Applications, New York, Columbus, 1963.
16. Braginskiy, V. B. and A. B. Manukin, *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 52, p. 988, 1967.
17. Robertson, H., *Monthly Notices*, Vol. 97, p. 423, 1937.
18. Radziyevskiy, V. V., *DAN* [*Doklady Akademii Nauk SSSR*], Vol. 74, p. 197, 1950.
19. Smiley, V. N., *Proc. IEEE*, No. 1, p. 158, 1963.
20. Landau, L. D. and Ye. M. Lifshits, *Statisticheskaya Fizika* [Statistical Physics], "Nauka" Press, 1964.
21. Gol'dman, I. I. and V. D. Krivchenkov, *Sbornik Zadach po Kvantovoy Mekhanike* [Collection of Articles on Problems in Quantum Mechanics], "Gostekhizdat" Press, 1957.
22. Eotvos, R. V., D. Pekar and E. Fekete, *Ann. d. Phys.*, Vol. 68, p. 11, 1922.

23. Aokolov, A. A. and I. M. Ternov, *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 25, p. 698, 1953.
24. Zaytsev, Yu. I., *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 50, p. 527, 1966.
25. Bershteyn, I. L., I. A. Andrononva and Yu. I. Zaytsev, *Izv. VUZov, "Radiofizika"*, Vol. 10, p. 59, 1967.
26. White, R., *J. Appl. Phys.*, Vol. 34, p. 2123, 1963.
27. Carone, E., *Appl. Phys. Lett.*, Vol. 4, p. 95, 1964.
28. de Maria, A. J., *PIEEE*, Vol. 52, p. 73, 1964.
29. Braginskiy, V. B. and V. N. Rudenko, *VMU* [*Vestnik Moskovskogo Universiteta*], *Ser. III*, No. 3, p. 128, 1966.
30. Blokhintsev, D. I., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 95, p. 73, 1964.
31. Ginzburg, V. L., *Astronautica Acta*, Vol. 12, No. 2, p. 136, 1966.
32. Dicke, R. H., *Sci. Amer.*, Vol. 205, No. 84, 1961; P. G. Roll, R. Krotkov, R. H. Dicke, *Ann. Phys.*, Vol. 26, p. 442, 1964.
33. Dabbs, J. W., J. A. Harvay, D. Paya and H. Horstman, *Phys. Rev.*, *Second Series*, Vol. 139, No. 3, B, 756, 1965.
34. Morgan, T. and A. Peres, *Phys, Rev. Lett.*, Vol. 9, p. 79, 1962.
35. Jeffries, A., *Dinamicheskaya oriyentatsiya yader* [Dynamic Orientation of Nuclei], "Mir" Press, 1966.
36. Braginskiy, V. B., L. I. Slabkiy and V. K. Martynov, *VMU* [*Vestnik Moskovskogo Universiteta*], *Ser. III*, p. 122, 1967.
37. Winen, A., *Phys. Rev. Lett.*, Vol. 1, p. 37, 1959.
38. Deaver, B. C. and W. M. Fairbank, *Phys. Rev. Lett.*, Vol. 7, p. 43, 1961.
39. Doll, R. and M. Näbauer, *Phys. Rev. Lett.*, Vol. 7, p. 51, 1961.
40. Pitayevskiy, L. P., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 90, p. 623, 1966.
41. Einstein, A. and W. de Haas, *Verhandl. deut. physik. Ges.*, Vol. 17, p. 152, 1915.
42. King, J. G., *Phys. Rev. Lett.*, Vol. 5, p. 562, 1960.
43. Gell-Mann, *Phys. Rev. Lett.*, Vol. 8, p. 214, 1964.
44. Zweig, G., preprint, *CERN*, 1964.
45. Zel'dovich, Ya. B., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 89, p. 647, 1966.
46. Gallinaro, G. and G. Morpurgo, preprint of 52nd Conference of Italian Phys. Soc., Oct. 1966.
47. Braginskiy, V. B., Ya. B. Zel'dovich, V. K. Martynov and V. V. Migulin, *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 52, p. 29, 1967.
48. Braginskiy, V. B., Ya. B. Zel'dovich, V. K. Martynov and V. V. Migulin, *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 54, p. 91, 1968.
49. Graginskiy, V. B., L. S. Korniyenko and S. S. Poloskov, *VMU* [*Vestnik Moskovskogo Universiteta*], *Ser. III*, p. 113, 1968.
50. Braunbeck, W., *Zs. Phys.*, Vol. 112, p. 764, 1939.
51. Chupka, W., J. Shiffer and C. Stevens, *Phys. Rev. Lett.*, Vol. 17, p. 60, 1966.
52. Stanyukovich, K. P., *Gravitatsinonnoye pole i elementarnyye chastitsy* [Gravitational Field and Elementary Particles], "Nauka" Press, 1965.
53. Mikhaylov, A. A., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol, 59, p. 51, 1956.

54. Landau, L. D. and Ye. M. Lifshits, *Teoriya polya* [Field Theory], "Fizmatgiz" Press, 1960.
55. Kopal, Z. and M. Shapley, "Catalogue of the Elements of Eclipsing Binary Systems", *Jodrell Bank Annals*, Vol. 1, p. 141, fasc. 4, 1956.
56. Kraft, R., J. Mathews and Y. Greenstein, *Astrophys. J.*, Vol. 136, p. 312, 1962; R. Kraft, *Astrophys. J.*, Vol. 139, p. 457, 1964.
57. Mironovskiy, V. N., *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 48, p. 358, 1965.
58. Zel'dovich, Ya. B. and I. D. Novikov, *DAN* [*Doklady Akademii Nauk SSSR*], Vol. 155, p. 1033, 1964.
59. Shklovskiy, I. S. and N. S. Kardashev, *DAN* [*Doklady Akademii Nauk SSSR*], Vol. 155, p. 1039, 1964.
60. Zel'dovich, Ya. B., Preprint *IMP AN SSSR*, 1967.
61. Dyson, F., Gravity Foundation Prize Essay, 1962.
62. Ivanenko, D. D. and A. A. Sokolov, *VMU* [*Vestnik Moskovskogo Universiteta*], *Ser. III*, No. 8, p. 103, 1947.
63. Wheeler, J. A. and D. Brill, *Rev. Mod. Phys.*, Vol. 29, p. 465, 1957.
64. Vladimirov, Yu. S., *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 45, p. 251, 1963.
65. de Sabbata, V., *Trudy 5-y Konferentsii po Teorii Otnositel'nosti i Gravitatsii* [Transactions of the 5th Conference on the Theory of Relativity and Gravitation], Tbilisi University Press, 1969.
66. Gandel'mann, G. M. and V. S. Pinayev, *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 37, p. 1073, 1953.
67. Gertsenshteyn, M. Ye., *ZhETF* [*Zhurnal Eksperimentalnoy i Teoreticheskoy Fiziki*], Vol. 41, p. 1904, 1962.
68. Weber, J., *Obshchaya Teoriya Otnositel'nosti i Gravitatsionnyye Volny* [General Theory of Relativity and Gravitational Waves], "Inostrannaya Literatura" Press, 1962.
69. Wheeler, J., *Gravitatsiya Neytrino i Vselennaya* [Gravitation Neutrinos and the Universe], "Inostrannaya Literature" Press, 1962.
70. Kopvillem, U. Kh., *Trudy 5-y Konferentsii po Teorii Otnositel'nosti i Gravitatsii* [Transactions of the 5th Conference on the Theory of Relativity and Gravitation], Tbilisi University Press, 1969.
71. AIAA, reports III meeting, January 1966.
72. Braginskiy, V. B., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 83, p. 433, 1965.
73. Weber, J., *Phys. Rev. Lett.*, Vol. 17, p. 1228, 1966.
74. Sinsky, J. and J. Weber, *Phys. Rev. Lett.*, Vol. 18, p. 795, 1967.
75. Weber, J., *Phys. Today*, Vol. 21, p. 34, 1968.
76. Weber, J., *Phys. Rev. Lett.*, Vol. 20, p. 1307, 1968.
77. Braginskiy, V. B. and V. N. Rudenko, *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 11, p. 395, 1970.
78. Shiff, L., *Trudy 5-y Konferentsii po Teorii Otnositel'nosti i Gravitatsii* [Transactions of the 5th Conference on the Theory of Relativity and Gravitation], Tbilisi University Press, 1969.
79. Everitt, C. W. F. and W. M. Fairbank, "Application of Low Temperature Techniques to a Satellite Test of General Relativity", Stanford University Preprint, 1968.

80. Shapiro, L. F., *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 95, p. 145, 1968.
81. Ignatovich, V. K., Preprint *OIYaI* [Joint Institute of Nuclear Research], 1968.
82. Schiff, L., *Phys. Rev.*, Vol. 132, p. 2194, 1963.
83. Sandars, P. G. H., *Phys. Lett.*, Vol. 14, p. 194, 1965; Vol. 22, p. 290, 1966.
84. Sakharov, A. D., Preprint *OIYaI* [Joint Institute of Nuclear Research], 1969.
85. North, J. C. and R. C. Buschert, *Rev. Sci. Instr.*, Vol. 37, p. 325, 1966.
86. Ho, J. T. and A. E. Siegman, *Transactions IRE, MTT-9*, p. 459, 1961.
87. Bershteyn, I. L., *Izv. AN SSSR, Ser. Fizika*, Vol. 24, p. 145, 1950.
88. Brenner, J., *Sverkhprovodyashchiye Ustroystva* [Superconducting Devices], "Mir" Press, 1964.
89. Braginskiy, V. B. and V. I. Panov, *PTE*, No. 5, p. 136, 1969.
90. Vasin, I. G., P. G. Pozdnyakov and M. I. Yaroslavskiy, *DAN* [*Doklady Akademii Nauk SSSR*], Vol. 119, p. 481, 1958.
91. Beams, J. W., R. D. Boule and P. E. Hexner, *Rev. Sci. Instr.*, Vol. 32, p. 645, 1961.
92. Tatarskiy, V. I., *Teoriya Fluktuatsionnykh Yavleniy pri Rasprostranenii Radiovoln v Turbulentnoy Atmosfere* [Theory of Fluctuation Phenomena in the Propagation of Radio Waves in the Turbulent Atmosphere], Academy of Sciences of the USSR Press, 1969.
93. Born, M. and A. Wolf, Principles of Optics, New York, 1959.
94. Bershteyn, I. L., *DAN* [*Doklady Akademii Nauk SSSR*], Vol. 101, p. 1734, 1954.
95. Bennett, R., Gaseous Optical Masers, 1961.
96. Troitskiy, V. S., *Radiotekhnika i Elektronika*, Vol. 1, p. 818, 1956.
97. *UFN* [*Uspekhi Fizicheskikh Nauk*], Vol. 92, p. 168, 1967.
98. De Gelli, D. and A. Rozar (eds.), *Kosmicheskaya Fizika* [Cosmic Physics], "Mir" Press, 1966.
99. Konstantinov, B. P., M. M. Bredov and Ye. P. Mazets, *DAN* [*Doklady Akademii Nauk SSSR*], Vol. 174, No. 3, p. 580, 1967.
100. Konstantinov, B. P., M. M. Bredov, Ye. P. Mazets, V. N. Panov, R. L. Aptekar', S. V. Golenitskiy, Yu. A. Gur'yan and V. N. Il'inskiy, "Micrometeor Investigations on the 'Kosmos-135' Satellite", Preprint, *FTI AN SSSR*, 1968.

Translated for the National Aeronautics and Space Administration under contract No. NASw-2037 by Techtran Corporation, P. O. Box 729, Glen Burnie, Maryland 21061.

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